Unmanned Systems, Vol. 13, No. 6 (2025) 1559–1568

DOI: 10.1142/S2301385025410080

UNMANNED SYSTEMS
https://www.worldscientific.com/worldscinet/us

Constrained Fencing Control of Networked Second-Order Nonlinear Agents for Multiple Targets via Adaptive Neurodynamic Optimization

Bin-Bin Hu*, Hai-Tao Zhang†

School of Artificial Intelligence and Automation,
The MOE Engineering Research Center of Autonomous Intelligent Unmanned Systems,
Huazhong University of Science and Technology,
Wuhan 430074, P. R. China

Guangdong HUST Industrial Technology Research Institute,
Guangdong Autonomous Unmanned Surface Vessel Engineering Technology Center,
Dongguan 523808, P. R. China

This paper develops a cooperative fencing control scheme of networked second-order nonlinear agents with multiple moving targets. A position-only distributed observer is designed to estimate the center state of the targets. Then, to fulfill state estimation, an optimal fencing controller is designed to achieve evenly distributed fencing of multiple targets via output feedback and adaptive neural optimization. Significantly, the asymptotical stability of the closed-loop system governed by the proposed collective fencing control law is guaranteed by the input-to-state (ISS) principle under nonlinear dynamics constraints. Finally, illustrative simulations are provided to show the effectiveness of the proposed fencing scheme.

Keywords: Constrained fencing control; neural optimization; multi-agent system.

1. Introduction

Over the years, a mainstream research theme of multi-agent systems (MASs) is coordinated formation [1–3], which has been widely explored by a large volume of studies during the past decades. Therein, a leaderless framework [4] was initially studied to address the regulation problem, which is essentially a consensus problem, as it involves regulating the agents to achieve agreement in certain states. For more sophisticated consensus tracking and following problems, the leader–follower framework [5] will be adopted, where the leader (i.e. target) is governed by prescribed trajectories. So far, most of the efforts are focused on only one leader. Regarding more complex multi-leader scenarios, one kind of representative work is the containment control [6], which regulates a group of followers to converge to a convex hull formed by the leaders. Conversely, fencing control acting as an inverse problem of the containment control is to drive a group of followers to fence (encircle) the leader(s) (i.e. targets) in a convex hull [7–9], which has drawn extensive attention for its application potential in collective convey, perimeter surveillance, collaborative escort, patrolling, and so on [10, 27].

Previously, most of the works concentrated on one target or target case with the easily available center of targets, where the target can be either stationary [11, 12] or motional [13, 14]. As a pioneering work, a

Received 18 June 2024; Accepted 9 May 2025; Published 15 July 2025.
This paper was recommended for publication in its revised form by Special Issues Editors, Hai-Tao Zhang, Chen Lyu and Bin-Bin Hu.
Email address: *hbb@hust.edu.cn, †zht@mail.hust.edu.cn
†Corresponding author.

limit-cycle-based strategy was developed to form a circular formation for the target [11]. Later on, a cooperative controller consisting of attractive, repulsive, and rotation components was designed to fence a specified target [12]. Unfortunately, stationary targets are generally impractical in real fencing applications, which has thus motivated more recent efforts devoted to fencing for motional targets. For instance, a distributed controller was developed to form a circle formation with direct topologies [13] and switching topologies [14], respectively.

Due to the more sophisticated environment and complex tasks, more researchers were attracted to study the fencing problem with multiple targets, where the core challenge lies in deriving the center state of targets by the followers. However, such a kind of central information may not be directly accessible to all followers, particularly when they can only reach a limited number of targets. As a representative work, Chen *et al.* [15] first addressed multiple stationary leaders enclosing problems under an estimation-and-control framework with connected topologies. Afterward, Shoja *et al.* [16] proposed an adaptive controller for networked nonlinear and heterogeneous agents. Unfortunately, the relevant existing works [15, 16] only consider first-order systems and have not touched higher-order dynamics, which is more practical in industrial unmanned systems, sensor networks, mechanical systems, etc. To this end, Shi *et al.* [17] presented a fencing controller for linear second-order MAS with the assistance of Lasalle's invariance principle. Li *et al.* [18] afterward extended the targets enclosing problem with a smooth estimator for the center of targets. Dong *et al.* developed a time-varying formation control protocol with multiple targets of either linear [19] or nonlinear dynamics [20], respectively. Moreover, Hu *et al.* [21] proposed an adaptive fencing controller to encircle multiple moving targets with unmanned surface vessel dynamics, which was afterward extended to the fencing control of scattered moving targets [22].

It is still worth mentioning that the nonlinear constraints form another research hotspot in MAS control, which intensifies the challenging evenly distributed fencing controller design. Till now, lots of efforts have been devoted to dealing with the nonlinear constraints in MAS formation. They can be accordingly classified into control protocols [23, 24] and optimization methods [25, 26]. However, due to the challenges in stability analysis of nonlinear coupling dynamics, it remains a dilemma to design a collective fencing control protocol for nonlinear MASs governed by nonlinear constraints.

To this end, we hereby seek to address the constrained fencing control problem of networked second-order nonlinear agents with multiple motional targets. The contribution is two-fold:

(1) Develop a position-only distributed observer for the center of multiple targets with disconnected topology among followers.
(2) Propose an adaptive fencing protocol for constrained nonlinear second-order agents to evenly fence multiple targets via output feedback and adaptive neural optimization, respectively.

The rest of this paper is organized as follows. Section 2 formulates the main problem. Section 3 designs a distributed observer and develops an evenly distributed fencing control scheme via output feedback. Section 4 conducts extensive simulations to verify the effectiveness of the present design. Conclusions are finally drawn in Sec. 5.

Throughout the paper, $\mathbb{R}, \mathbb{R}^+$ denote the real and positive real numbers, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\| * \|$ is the Euclidean norms, $1_m = [1, \dots, 1]^\top \in \mathbb{R}^m$, $I_n \in \mathbb{R}^n$ is the identity matrix. $(*)_{i,j}$ represents the (i,j)th entry of matrix $(*)$, $\lambda_{\min}(*)$ represents the smallest eigenvalue of matrix $(*)$, and $\nabla(f)$ is the gradient of function f. $\otimes$ denotes the Kronecker product.

2. Problem Formulation

We consider an MAS with $n + m$ agents, where the communication topology is denoted to be $G(\nu, \varepsilon)$ with the vertex set $\nu = [\nu_1, \nu_2, \dots, \nu_{n+m}]$ and the edge set $\varepsilon = \nu \times \nu$. $A = [a_{i,j}] \in \mathbb{R}^{(n+m)\times(n+m)}$ is the adjacency matrix, where $a_{i,j} = 1, i \neq j$, if $(\nu_i, \nu_j) \in \varepsilon$, and $a_{i,j} = 0$ otherwise. Self-loop is not allowed, i.e. $a_{i,i} = 0$. $L = [l_{i,j}] \in \mathbb{R}^{(n+m)\times(n+m)}$ denotes the Laplacian matrix, where $l_{i,i} = \sum_{i=1, i\neq j}^{n+m} a_{i,j}$ and $l_{i,j} = -a_{i,j}, i \neq j$.

Here, $G(\nu, \varepsilon)$ of the MAS can be divided into followers and targets, where the vertex i is a follower if it has neighbor(s) and a target otherwise. Let $\mathcal{F} = [\nu_1, \nu_2, \dots, \nu_n]$ and $\mathcal{T} = [\nu_{n+1}, \nu_{n+2}, \dots, \nu_{n+m}]$ denote the follower and target subsets, respectively, without generality. To distinguish the follower group and target group, we hereby separate the matrix L into two parts, namely,

$$L = \begin{bmatrix} L_1 & L_2 \\ \mathbf{0}_{m\times n} & \mathbf{0}_{m\times m} \end{bmatrix}, \tag{1}$$

where $L_1 \in \mathbb{R}^{n\times n}$, $L_2 \in \mathbb{R}^{n\times m}$, $\mathbf{0}_{m\times n} \in \mathbb{R}^{m\times n}$, and $\mathbf{0}_{m\times m} \in \mathbb{R}^{m\times m}$ represent the zero matrices.

The agent's dynamic can be described to be

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = f_i(v_i) + u_i(t) + d_i(t), \quad i \in \mathcal{F}, \\ \dot{r}_k(t) = v_d, k \in \mathcal{T}, \end{cases} \tag{2}$$

where $x_i(t) \in \mathbb{R}^2, v_i(t) \in \mathbb{R}^2, r_k(t), v_d \in \mathbb{R}^2$ denote the positions and velocities of follower i and target k, respectively; $f_i(v_i) \in \mathbb{R}^2$ is an nonidentical nonlinear function concerning with the velocity v_i; $u_i(t) \in \mathbb{R}^2, u_{\min} \leq u_i \leq u_{\max}$ denotes the constrained control input with $u_{\min}, u_{\max}$ being the constant values; $d_i(t) \in \mathbb{R}^2, \|d_i\| \leq b_i$, is a bounded external disturbance. Here, the agent dynamics are commonly used in the literature, such as unmanned ground vehicles (UGVs) and unmanned surface vessels (USVs) [27–29].

To proceed further investigation, a definition of well-informed and uninformed followers is introduced first.

Definition 2.1 ([19]). The follower i is defined well-informed if $\sum_{k=n+1}^{n+m} a_{i,k} = m$, and uninformed if $\sum_{k=n+1}^{n+m} a_{i,k} = 0$, where $\alpha_{i,k}$ are the elements of the adjacency matrix A.

In practice, the well-informed followers are usually equipped with communication devices and advanced sensors such as vision cameras, laser radars, and millimeter wave radars to obtain information about all targets, whereas uninformed followers are only equipped with communication devices to obtain information about neighboring followers, which reduces the equipment cost.

With Definition 2.1, an assumption on communication topology $G(\nu, \varepsilon)$ is necessary.

Assumption 2.2. There only exist well-informed and uninformed followers in the target-follower MAS, and for each uninformed follower, there exists a directed path to at least one well-informed follower.

Assumption 2.2 indicates that the submatrix L_1 in (1) is positive definite, i.e. $L_1 \succ 0$ because there exists at least one well-informed follower, i.e.

Lemma 2.3 ([19]). *Under Assumption* 2.2, *one has that each entry of* $-L_1^{-1}L_2$ *is nonnegative and all the rows of* $-L_1^{-1}L_2$ *are identical that equal* $[a_{i,n+1}, \ldots, a_{i,n+m}]/\sum_{k=n+1}^{n+m} a_{i,k}$, *where* $a_{i,k} > 0$, $i \in \mathcal{F}$, $k \in \mathcal{T}$, *and the matrix* L_1, L_2 *are given* (1).

Definition 2.4 ([17] (Equal fencing)). Let $\eta_i = \rho[\cos 2\pi i/n, \sin 2\pi i/n]^\top$ be the relative fencing vector with the fencing radius $\rho > \max\|r_j - r_k\|, j, k \in \mathcal{T}$, one has that the equal fencing for the follower group $\mathcal{F}$ is achieved if

$$\lim_{t\to\infty} \|x_i(t) - \bar{r}(t) - \eta_i\| \leq \sigma, \quad i \in \mathcal{F}, \tag{3}$$

with $\bar{r} := \sum_{k=n+1}^{n+m} r_k/m$ and $\sigma > 0$.

It is worth mentioning that ρ is available with a well-informed follower in Definition 2.4, which fulfills the goal of fencing the convex hull $\mathcal{C}o(\mathcal{T}) = \sum_{k=n+1}^{n+m} \lambda_k r_k, \lambda_k \geq 0$ (see e.g. [15]) by follower set $\mathcal{F}$. However, the velocity information is always unavailable in practice and intensifies the difficulties of designing a controller merely with position information, which thus motivates the problem addressed in this paper.

Problem 1. Design a cooperative controller $u_i = g(x_i, f_i, d_i, r_k), i \in \mathcal{F}, k \in \mathcal{T}$, via an output feedback for the target-follower MAS governed by (2) to achieve the equal fencing in Definition 2.4.

3. Main Results

In this section, the design and stability analysis of coordinated equal fencing is divided into three parts, namely, a distributed observer for the center of target set $\mathcal{T}$, a fencing controller via output feedback, and convergence analysis, respectively.

3.1. *Distributed observer of targets' center*

Let $\hat{r}_i(t) \in \mathbb{R}^2, \hat{v}_d^i(t) \in \mathbb{R}^2, i \in \mathcal{F}$, be the estimators of follower i for $\bar{r}, v_d$ in (2) and (3), respectively, one has that

$$\begin{cases} \dot{\hat{r}}_i(t) = -k\left[\sum_{j=1}^{n} a_{i,j}(\hat{r}_i(t) - \hat{r}_j(t)) \right. \\ \qquad \left. + \sum_{k=n+1}^{n+m} a_{i,k}(\hat{r}_i(t) - r_k(t))\right] + \hat{v}_d^i(t), \\ \dot{\hat{v}}_d^i(t) = -\alpha k\left[\sum_{j=1}^{n} a_{i,j}(\hat{r}_i(t) - \hat{r}_j(t)) \right. \\ \qquad \left. + \sum_{k=n+1}^{n+m} a_{i,k}(\hat{r}_i(t) - r_k(t))\right], \end{cases} \tag{4}$$

where $k \in \mathbb{R}^+, \alpha \in \mathbb{R}^+$ are observer gains of follower i.

Lemma 3.1. *Under Assumption* 1, *the observer* $\hat{r}_i(t)$, $\hat{v}_d^i(t), i \in \mathcal{F}$, *governed by* (4) *asymptotically converges to the center state of target set* $\mathcal{T}$, *i.e.*

$$\lim_{t\to\infty} \hat{r}_i(t) - \bar{r}(t) = 0, \quad \lim_{t\to\infty} \hat{v}_d^i(t) = v_d,$$

with $\bar{r}$ *and* v_d *given in* (2) *and* (3), *respectively.*

Proof. First, we define the observed position error $\tilde{x}_i \in \mathbb{R}^2$ and velocity error $\tilde{v}_i \in \mathbb{R}^2$ to be

$$\begin{cases} \tilde{r}_i := \hat{r}_i - \bar{r}, \\ \tilde{v}_d^i := \hat{v}_d^i - v_d. \end{cases} \tag{5}$$

Taking the derivative of (5) and substituting (4) into it yields

$$\begin{cases} \dot{\tilde{r}}_i(t) = -k\left[\sum_{j=1}^{n} a_{i,j}(\tilde{r}_i(t) - \tilde{r}_j(t)) \right. \\ \qquad \left. + \sum_{k=n+1}^{n+m} a_{i,k}(\tilde{r}_i + \bar{r}(t) - r_k(t))\right] + \tilde{v}_d^i, \\ \dot{\tilde{v}}_d^i(t) = -\alpha k\left[\sum_{j=1}^{n} a_{i,j}(\tilde{r}_i(t) - \tilde{r}_j(t)) \right. \\ \qquad \left. + \sum_{k=n+1}^{n+m} a_{i,k}(\tilde{r}_i + \bar{r}(t) - r_k(t))\right]. \end{cases} \tag{6}$$

Let $\tilde{r} = [\tilde{r}_1^\top, \tilde{r}_2^\top, \dots, \tilde{r}_n^\top]^\top \in \mathbb{R}^{2n}, \tilde{v}_d = [(\tilde{v}_d^1)^\top, (\tilde{v}_d^2)^\top, \dots, (\tilde{v}_d^n)^\top]^\top \in \mathbb{R}^{2n}, r = [r_1^\top, r_2^\top, \dots, r_m^\top]^\top \in \mathbb{R}^{2n}$. According to Lemma 2.3, one has

$$\begin{cases} \dot{\tilde{r}}(t) = -k[L_1 \otimes I_2(\tilde{r} - L_1^{-1}L_2 \otimes I_2 r) \\ \qquad + L_2 \otimes I_2 r] + \tilde{v}_d \\ \quad = -k(L_1 \otimes I_2)\tilde{r} + \tilde{v}_d, \\ \dot{\tilde{v}}_d(t) = -\alpha k(L_1 \otimes I_2(\tilde{r} - L_1^{-1}L_2 \otimes I_2 r) \\ \qquad + L_2 \otimes I_2 r) \\ \quad = -\alpha k(L_1 \otimes I_2)\tilde{r}. \end{cases} \tag{7}$$

Let $\sigma := [\tilde{r}^\top, \tilde{v}_d^\top] \in \mathbb{R}^{4n}$, it follows from (7) that $\dot{\sigma} = A\sigma$ with

$$A = \begin{bmatrix} -kL_1 \otimes I_2 & I_n \otimes I_2 \\ -\alpha k L_1 \otimes I_2 & \mathbf{0} \end{bmatrix} \in \mathbb{R}^{4n \times 4n}.$$

Recalling $L_1 \succ 0$ below Assumption 2.2, one has that A is a Hurwitz matrix, which implies that there exists a positive definite matrix $P \succ 0$ such that $A^\top P + PA = -\epsilon I_{4n}$ with $\epsilon \in \mathbb{R}^+$.

Consider a Lyapunov function candidate $V(\sigma) = \sigma^\top P\sigma/2$, whose derivative is

$$\dot{V}(\sigma) = \sigma^\top(A^\top P + PA)\sigma = -\epsilon\|\sigma\|^2 \le 0,$$

which implies that σ converges to zero as $t \to \infty$, i.e. $\lim_{t\to\infty} \sigma(t) = 0$. Equivalently, the observers $\hat{r}_i(t), \hat{v}_d^i(t), i \in \mathcal{F}$, finally converge to the center of the target set $\mathcal{T}$. □

Remark 3.2. The observers $\hat{r}_i(t), \hat{v}_d^i(t), i \in \mathcal{F}$, only leverage position information of target set L, which implies that fewer sensors are necessarily equipped.

Remark 3.3. The communications among followers are not necessarily connected and undirected [15–18]. Furthermore, when there are two or more well-informed followers, the communication topology among followers can be disconnected because these followers have access to all the targets, which thereby can substantially reduce the communication burden among the followers.

3.2. *Fencing controller design*

To proceed with, let $\theta_i(v_i) \in \mathbb{R}^c$ to be the radial basis functions (RBFs) and $\omega_i \in \mathbb{R}^c$ the optimal parameters with the integer $c \in \mathbb{Z}^+$ (see e.g. [30]), one has that the nonlinear term $f_i(v_i)$ in Eq (2) is parameterized on a compact set $\Omega(v_i)$,

$$\begin{gathered} f_i(v_i) = \omega_i^\top\theta_i(v_i) + \epsilon_i, \\ \sup_{v_i\in\Omega(v_i)} \|\epsilon_i\| \le \epsilon_M, \|\omega_i\| \le \omega_M, \quad i \in \mathcal{F}, \end{gathered} \tag{8}$$

with the bias term ϵ_i, and the boundary $\epsilon_M > 0$ and $\omega_M > 0$. Let $\widehat{\omega}_i \in \mathbb{R}^c$ be an estimated parameter vector, and the estimation of $f(v_i)$ is described to be

$$\hat{f}_i(v_i) = \widehat{\omega}_i^\top\theta_i(v_i), \tag{9}$$

where $\theta_i(v_i)$ is abbreviated to be θ_i for simplicity. To further investigate the feasibility of the fencing controller, the following assumption is necessarily made.

Assumption 3.4. The upper boundaries ϵ_M, ω_M for $\|\epsilon_i\|, \|\omega_i\|, i \in \mathcal{F}$ are available to each follower $i, i \in \mathcal{F}$.

Then, we define the fencing position error $e_{i1} \in \mathbb{R}^2$ to be

$$e_{i1}(t) := x_i(t) - \hat{r}_i(t) - \eta_i, \quad i \in \mathcal{F}, \tag{10}$$

where $\hat{r}_i(t)$, η_i are given in (4) and Definition 2.4, respectively. Substituting (2) and (5) into (10) yields

$$\dot{e}_{i1} = v_i - v_d - \dot{\tilde{r}}_i. \tag{11}$$

To stabilize e_{i1}, a virtual signal $\alpha_i^d \in \mathbb{R}^2$ and filter-passed signal $\alpha_i^c \in \mathbb{R}^2$ for follower i are defined to be

$$\begin{gathered} \alpha_i^d := -k_{i1}e_{i1} + \hat{v}_d^i, \\ c_1\dot{\alpha}_i^c + \alpha_i^c := \alpha_i^d, \quad \alpha_i^c(0) = \alpha_i^d(0), \end{gathered} \tag{12}$$

where $k_{i1} \in \mathbb{R}^+$ is the control gain and $c_1 \in \mathbb{R}^+$ is the time constant. Combining (11) and (12) together yields

$$\dot{e}_{i1} = -k_{i1}e_{i1} + e_{i2} + (\alpha_i^c - \alpha_i^d) - \dot{\tilde{r}}_i + \tilde{v}_d^i, \tag{13}$$

with $e_{i2} := v_i - \alpha_i^c \in \mathbb{R}^2$ being the velocity error and $\tilde{v}_d^i$ given in (5). It follows from (2), (12), (13) that

$$\begin{aligned} \dot{e}_{i1} &= -k_{i1}e_{i1} + e_{i2} + (\alpha_i^c - \alpha_i^d) - \dot{\tilde{r}}_i + \tilde{v}_d^i, \\ \dot{e}_{i2} &= u_i + \omega_i^\top\theta_i + \epsilon_i + d_i - \dot{\alpha}_i^c. \end{aligned} \tag{14}$$

To eliminate the effects of the error between filtered signals $(\alpha_i^c - \alpha_i^d)$ in (14), the compensated signals $z_{i1} \in \mathbb{R}^2$,

$z_{i2} \in \mathbb{R}^2,\ i \in \mathcal{F}$, are introduced [31]

$$\begin{aligned} \dot{z}_{i1} &= -k_{i1}z_{i1} + z_{i2} + (\alpha_i^c - \alpha_i^d), \\ \dot{z}_{i2} &= -k_{i2}z_{i2} - z_{i1}. \end{aligned} \tag{15}$$

Let $\varpi_{i1} := e_{i1} - z_{i1} \in \mathbb{R}^2, \varpi_{i2} := e_{i2} - z_{i2} \in \mathbb{R}^2$ be the compensated errors, it follows from (14) and (15) that

$$\begin{aligned} \dot{\varpi}_{i1} &= -k_{i1}\varpi_{i1} + \varpi_{i2} - \dot{\tilde{r}}_i + \tilde{v}_i^d, \\ \dot{\varpi}_{i2} &= k_{i2}z_{i2} + z_{i1} + u_i + \omega_i^\top\theta_i + \epsilon_i + d_i - \dot{\alpha}_i^c. \end{aligned} \tag{16}$$

However, since the velocity v_i and the acceleration $\dot{v}_i$ of follower i are not always available induced by the unknown f_i and external disturbance d_i, an additional estimator $\hat{x}_i(t) \in \mathbb{R}^2, \hat{v}_i(t) \in \mathbb{R}^2$ using a serial–parallel structure [32] is designed for $x_i(t), v_i(t)$,

$$\begin{aligned} \dot{\hat{x}}_i &= -c_{i1}\tilde{x}_i + \hat{v}_i, \\ \dot{\hat{v}}_i &= -c_{i2}\tilde{x}_i + \widehat{\omega}_i^\top\theta_i(\hat{\xi}_i) + u_i, \end{aligned} \tag{17}$$

where $c_{i1}, c_{i2} \in \mathbb{R}^+, \hat{\xi}_i = [\hat{x}_i^\top, \hat{v}_i^\top, t]^\top$, and $\tilde{x}_i := \hat{x}_i - x_i \in \mathbb{R}^2$ is the estimated position error, $\widehat{\omega}_i$ is the estimated parameter vector given in (9).

Let $\tilde{v}_i := \hat{v}_i - v_i \in \mathbb{R}^2$ be the estimated velocity error, it follows from (2) and (17) that

$$\begin{aligned} \dot{\tilde{x}}_i &= -c_{i1}\tilde{x}_i + \tilde{v}_i, \\ \dot{\tilde{v}}_i &= -c_{i2}\tilde{x}_i + \widetilde{\omega}_i^\top\theta_i(\hat{\xi}_i) + \zeta_i - \epsilon_i - d_i, \end{aligned} \tag{18}$$

with $\zeta_i = \omega_i^\top\theta_i(\hat{\xi}_i) - \omega_i^\top\theta_i(v_i)$ satisfying the following assumption. Let $\mathcal{Z}_i = [\tilde{x}_i^\top, \tilde{v}_i^\top]^\top \in \mathbb{R}^4$, one has that (18) becomes

$$\dot{\mathcal{Z}}_i = A_i\mathcal{Z}_i + B(\widetilde{\omega}_i^\top\theta_i(\hat{\xi}_i) + \zeta_i - \epsilon_i - d_i), \quad i \in \mathcal{F}, \tag{19}$$

with

$$A_i = \begin{bmatrix} -c_{i1} & 0 & 1 & 0 \\ 0 & -c_{i1} & 0 & 1 \\ -c_{i2} & 0 & 0 & 0 \\ 0 & -c_{i2} & 0 & 0 \end{bmatrix} \in \mathbb{R}^{4\times4},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{4\times2}.$$

Define $\hat{e}_{i2} := \hat{v}_i - \alpha_i^c \in \mathbb{R}^2$ to be the modified fencing velocity error, it follows from (14) and (18) that the derivatives of $e_{i1}, \hat{e}_{i2}$ rewrites

$$\begin{aligned} \dot{e}_{i1} &= -k_{i1}e_{i1} + \hat{e}_{i2} + (\alpha_i^c - \alpha_i^d) - \dot{\tilde{r}}_i + \tilde{v}_d^i - \tilde{v}_i, \\ \dot{\hat{e}}_{i2} &= -c_{i2}\tilde{x}_i + \widehat{\omega}_i^\top\theta_i(\hat{\xi}_i) + u_i - \dot{\alpha}_i^c, \end{aligned} \tag{20}$$

with $\tilde{v}_d^i, \tilde{v}_i, \alpha_i^c, \alpha_i^d$ given (5) and (12), respectively. Comparing with the compensated signals in (15), one has that the derivatives of the modified compensated errors in (16) become

$$\begin{aligned} \dot{\varpi}_{i1} &= -k_{i1}\varpi_{i1} + \widehat{\varpi}_{i2} - \dot{\tilde{r}}_i + \tilde{v}_d^i - \tilde{v}_i, \\ \dot{\widehat{\varpi}}_{i2} &= k_{i2}z_{i2} + z_{i1} - c_{i2}\tilde{x}_i + \widehat{\omega}_i^\top\theta_i(\hat{\xi}_i) + u_i - \dot{\alpha}_i^c, \end{aligned} \tag{21}$$

with $\widehat{\varpi}_{i2} := \hat{e}_{i2} - z_{i2}$. To this end, an adaptive fencing controller with output feedback is proposed as follows:

$$\begin{aligned} u_i^c &= -k_{i2}\hat{e}_{i2} - e_{i1} - \widehat{w}_i^\top\theta_i(\hat{\xi}_i) + \dot{\alpha}_i^c, \\ \dot{\widehat{\omega}}_i &= \gamma_i[-\tilde{x}_i^\top\theta_i(\hat{\xi}_i) - k_{i3}\widehat{\omega}_i], \end{aligned} \tag{22}$$

where the control gain $\gamma_i \in \mathbb{R}^+$ is a positive constant.

3.3. *Convergence analysis*

In this section, we will present a detailed convergence analysis of the closed-loop system governed by the controller (22). To proceed with, a necessary assumption concerning the upper boundaries of $\zeta_i, \theta_i(\hat{\xi}_i)$ is required.

Assumption 3.5. There exist upper bounds $\zeta_M > 0$, $\theta_M > 0$, such that for any follower $i, i \in \mathcal{F}$, $\|\zeta_i\| \leq \zeta_M$, $\|\theta_i(\hat{\xi}_i)\| \leq \theta_M$.

Assumption 3.5 is necessary and reasonable, which will be utilized in the proof of Lemma 3.6 later.

Lemma 3.6. *For the closed-loop system governed by the error dynamic* (19), *the compensated error* (21), *the adaptive controller* (22) *under Assumptions* 2.2, 3.4, 3.5 *with the input signals $\epsilon_i, l_i, \widetilde{u}_i, \omega_i, d_i, \dot{\tilde{x}}_i$, and $\tilde{v}_i$ are input-to-state* (*ISS*) *stable* [33] *provided that the parameters satisfy*

$$k_{i1} - \frac{3}{2} > 0, \quad k_{i2} - \frac{c_{i2}}{2} - 1 > 0, \quad \frac{k_{i3} - \theta_M^2}{2} > 0.$$

$$Q_i + \frac{1}{2}(P_iA_i + A_i^TP_i + (P_iB - C)(P_iB - C)^T + 3P_iBB^TP_i + \mathcal{K}_i) \leq 0,$$

where $P_i \in \mathbb{R}^{2\times2} \succ 0, Q_i \in \mathbb{R}^{2\times2} \succ 0, i \in \mathcal{F}$, are positive definite matrixes and

$$C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathcal{K}_i = \begin{bmatrix} c_{i2} & 0 \\ 0 & 1 \end{bmatrix} \succ 0. \tag{23}$$

Proof. Since the optimal parameters $\omega_i \in \mathbb{R}^c$ in (8) are constants, it follows from (22) that

$$\dot{\tilde{\omega}}_i = \gamma_i[-\tilde{x}_i^\top\theta_i(\hat{\xi}_i) - k_{i3}\widehat{\omega}_i]. \tag{24}$$

Substituting (22) into (21) yields

$$\begin{aligned} \dot{\varpi}_{i1} &= -k_{i1}\varpi_{i1} + \widehat{\varpi}_{i2} - \dot{\tilde{r}}_i + \tilde{v}_d^i - \tilde{v}_i, \\ \dot{\widehat{\varpi}}_{i2} &= -k_{i2}\widehat{\varpi}_{i2} - \varpi_{i1} - c_{i2}\tilde{x}_i + l_i + \widetilde{u}_i. \end{aligned} \tag{25}$$

Define a symmetrical matrix $P_i \succ 0 \in \mathbb{R}^4$, and pick a Lyapunov function candidate $V(t) = V_1(t) + V_2(t) + V_3(t)$ with

$$V_1(t) = \frac{1}{2}\sum_{i=1}^{n} \mathcal{Z}_i^\top P_i \mathcal{Z}_i, \quad V_2(t) = \frac{1}{2}\sum_{i=1}^{n} \widetilde{\omega}_i^\top \gamma_i^{-1} \widetilde{\omega}_i,$$
$$V_3(t) = \frac{1}{2}\sum_{i=1}^{n} \{\varpi_{i1}^\top \varpi_{i1} + \widehat{\varpi}_{i2}^\top \widehat{\varpi}_{i2}\}, \tag{26}$$

with $\mathcal{Z}_i, \widetilde{\omega}_i, \varpi_{i1}, \widehat{\varpi}_{i2}$ given in (19), (24), and (21), respectively. Combining (19), (21), and (25) yields the derivative of $V_i(t)$, $i = 1, 2, 3$ fulfilling

$$\dot{V}_1(t) = \sum_{i=1}^{n} \Big\{ \frac{1}{2}\mathcal{Z}_i^\top (P_i A_i + A_i^\top P_i) Z_i + \mathcal{Z}_i^\top P_i B \widetilde{\omega}_i^\top \theta_i(\hat{\xi}_i) + \mathcal{Z}_i^\top P_i B \zeta_i - \mathcal{Z}_i^\top P_i B^\top \epsilon_i - \mathcal{Z}_i^\top P_i B^\top d_i) \Big\},$$

$$\dot{V}_2(t) \le \sum_{i=1}^{n} \Big\{ -\widetilde{\omega}_i^\top \mathcal{Z}_i^\top C \theta_i(\hat{\xi}_i) - \frac{k_{i3}}{2}\widetilde{\omega}_i^\top \widetilde{\omega}_i + \frac{k_{i3}}{2}\omega_i^\top \omega_i \Big\},$$

$$\dot{V}_3(t) \le \sum_{i=1}^{n} \Big\{ -\Big(k_{i1} - \frac{3}{2}\Big)\varpi_{i1}^\top \varpi_{i1} - \Big(k_{i2} - \frac{c_{i2}}{2} - 1\Big)\widehat{\varpi}_{i2}^\top \widehat{\varpi}_{i2} + \frac{1}{2}\mathcal{Z}_i^\top \mathcal{K}_i \mathcal{Z}_i + \frac{1}{2}(\dot{\tilde{r}}_i^\top \dot{\tilde{r}}_i + (\tilde{v}_d^i)^\top \tilde{v}_d^i + l_i^\top l_i + \tilde{u}_i^\top \tilde{u}_i) \Big\}.$$

Utilizing the inequalities,

$$\mathcal{Z}_i^\top (P_i B - C_i)\widetilde{\omega}_i^\top \theta_i(\hat{\xi}_i) \le \frac{\mathcal{Z}_i^\top (P_i B - C_i)(P_i B - C_i)^\top \mathcal{Z}_i}{2} + \frac{\theta_M^2}{2}\widetilde{\omega}_i^\top \widetilde{\omega}_i,$$
$$\mathcal{Z}_i^\top P_i B(\zeta_i - \epsilon_i - d_i) \le \frac{3}{2}(\mathcal{Z}_i^\top P_i B B^\top P_i \mathcal{Z}_i) + \frac{(\zeta_i^\top \zeta_i + \epsilon_i^\top \epsilon_i + d_i^\top d_i)}{2},$$

with assistance of Assumption 3.5, one has

$$\dot{V}(t) \le \sum_{i=1}^{n} \Big\{ \frac{1}{2}\mathcal{Z}_i^\top (P_i A_i + A_i^\top P_i + (P_i B - C_i) \times (P_i B - C_i)^\top + 3 P_i B B^\top P_i + \mathcal{K}_i)\mathcal{Z}_i - \Big(k_{i1} - \frac{3}{2}\Big)\|\varpi_{i1}\|^2 - \Big(k_{i2} + \frac{c_{i2}}{2} - 1\Big) \times \|\widehat{\varpi}_{i2}\|^2 - \Big(\frac{k_{i3} - \theta_M^2}{2}\Big)\|\widetilde{\omega}_i\|^2 \Big\} + \xi,$$

with $\xi = \sum_{i=1}^{n}\{\|\zeta_i\|^2/2 + \|\epsilon_i\|^2/2 + \|d_i\|^2/2 + k_{i3}\|\omega_i\|^2/2 + \|l_i\|^2/2 + \|\tilde{u}_i\|^2/2 + \|\dot{\tilde{r}}_i\|^2/2 + \|\tilde{v}_d^i\|^2/2\}$. Define

$$b_{i1} := k_{i1} - \frac{3}{2} > 0,$$
$$b_{i2} := k_{i2} - \frac{c_{i2}}{2} - 1 > 0,$$
$$b_{i3} := \frac{(k_{i3} - \theta_M^2)}{2} > 0,$$

one has that

$$Q_i + \frac{1}{2}(P_i A_i + A_i^\top P_i + (P_i B - C_i)(P_i B - C_i)^\top + 3 P_i B B^\top P_i + \mathcal{K}_i) \preceq 0,$$

with $Q_i \succ 0 \in \mathbb{R}^4$ a positive definite matrix, and define $E_i = [\mathcal{Z}_i^\top, \omega_i^\top, \varpi_{i1}^\top, \widehat{\varpi}_{i2}^\top]^\top \in \mathbb{R}^8$ and $b_{iM} = \min\{\lambda_{\min}(Q_i), b_{i1}, b_{i2}, b_{i3}\}$ with $\lambda_{\min}(Q_i)$ being the smallest eigenvalue of the matrix Q_i, one has

$$\dot{V}(t) \le \sum_{i=1}^{n} \Big\{ -\frac{biM}{2}\|E_i\|^2 - \Big(\frac{biM}{2}\|E_i\|^2 - \frac{\|\zeta_i\|^2}{2} - \frac{\|\epsilon_i\|^2}{2} - \frac{\|d_i\|^2}{2} - \frac{k_{i3}\|\omega_i\|^2}{2} - \frac{\|l_i\|^2}{2} - \frac{\|\tilde{u}_i\|^2}{2} - \frac{\|\dot{\tilde{r}}_i\|^2}{2} - \frac{\|\tilde{v}_d^i\|^2}{2} \Big\}. \tag{27}$$

From the fact that

$$\|E_i\| \ge \frac{\|\zeta_i\|}{\sqrt{b_{iM}}} + \frac{\|\epsilon_i\|}{\sqrt{b_{iM}}} + \frac{\|d_i\|}{\sqrt{b_{iM}}} + \frac{\sqrt{k_{i3}}\|\omega_i\|}{\sqrt{b_{iM}}} + \frac{\|l_i\|}{\sqrt{b_{iM}}} + \frac{\|\tilde{u}_i\|}{\sqrt{b_{iM}}} + \frac{\|\dot{\tilde{r}}_i\|}{\sqrt{b_{iM}}} + \frac{\|\tilde{v}_d^i\|}{\sqrt{b_{iM}}} \ge \varsigma_i,$$

with $\varsigma_i = \sqrt{\frac{\|\zeta_i\|^2 + \|\epsilon_i\|^2 + \|d_i\|^2 + k_{i3}\|\omega_i\|^2 + \|l_i\|^2 + \|\tilde{u}_i\|^2 + \|\dot{\tilde{r}}_i\|^2 + \|\tilde{v}_d^i\|^2}{b_{iM}}}$, one has that

$$\dot{V}(t) \le -\sum_{i=1}^{n} \frac{biM}{2}\|E_i\|^2 \le 0.$$

Accordingly, there exists a $\mathcal{KL}$ function $\beta_2(\cdot)$ and $\gamma^{\zeta_i}(\cdot)$, $\gamma^{\epsilon_i}(\cdot)$, $\gamma^{d_i}(\cdot)$, $\gamma^{\omega_i}(\cdot)$, $\gamma^{l_i}(\cdot)$, $\gamma^{\tilde{u}_i}(\cdot)$, $\gamma^{\dot{\tilde{r}}_i}(\cdot)$, $\gamma^{\tilde{v}_d^i}(\cdot)$ satisfying

$$\|E_i(t)\| \le \beta_2(E_i(t_0), t - t_0) + \gamma^{\zeta_i}(\|\zeta_i\|) + \gamma^{\epsilon_i}(\|\epsilon_i\|) + \gamma^{d_i}(\|d_i\|) + \gamma^{\omega_i}(\|\omega_i\|) + \gamma^{l_i}(\|l_i\|) + \gamma^{\tilde{u}_i}(\|\tilde{u}_i\|) + \gamma^{\dot{\tilde{r}}_i}(\|\dot{\tilde{r}}_i\|) + \gamma^{\tilde{v}_d^i}(\|\tilde{v}_d^i\|), \tag{28}$$

with $\gamma^{\zeta_i}(s) = s/\sqrt{b_{iM}}$, $\gamma^{\epsilon_i}(s) = s/\sqrt{b_{iM}}$, $\gamma^{d_i}(s) = s/\sqrt{b_{iM}}$, $\gamma^{\omega_i}(s) = s\sqrt{k_{i3}}/\sqrt{b_{iM}}$, $\gamma^{\tilde{u}_i}(s) = s/\sqrt{b_{iM}}$, $\gamma^{l_i}(s) = s/\sqrt{b_{iM}}$, $\gamma^{\dot{\hat{r}}_i}(s) = s/\sqrt{b_{iM}}$, and $\gamma^{\tilde{v}_d^i}(s) = s/\sqrt{b_{iM}}$. □

Theorem 3.7. *For the closed-loop system governed by* (2) *and* (22), *under Assumptions* 2.2, 3.4, *and* 3.5, *Problem* 1 *is solved.*

Proof. Since $\beta_2(\cdot)$ approaches to zero when $t \to \infty$, it follows from (28) in Lemma 3.6 that the limits of $\|E_i(t)\|$ are bounded by

$$\begin{aligned}\|E_i(t)\| \le{}& \gamma^{\zeta_i}(\|\zeta_i\|) + \gamma^{\epsilon_i}(\|\epsilon_i\|) + \gamma^{d_i}(\|d_i\|) \\ &+ \gamma^{\omega_i}(\|\omega_i\|) + \gamma^{l_i}(\|l_i\|) + \gamma^{\tilde{u}_i}(\|\tilde{u}_i\|) \\ &+ \gamma^{\dot{\hat{r}}_i}(\|\dot{\hat{r}}_i\|) + \gamma^{\tilde{v}_d^i}(\|\tilde{v}_d^i\|),\end{aligned}$$

as $t \to \infty$. Combining with $\zeta_i \le \zeta_M$ in Assumption 3.5 yields

$$\begin{aligned}\|E_i(t)\| \le{}& \frac{\zeta_M}{\sqrt{b_{iM}}} + \frac{(\sqrt{\gamma_{zi2}+1}\epsilon_M)}{\sqrt{b_{iM}}} + \frac{\sqrt{k_{i3}}\omega_M}{\sqrt{b_{iM}}} \\ &+ \frac{\sqrt{\gamma_{zi2}}b_i}{\sqrt{b_{iM}}} + \gamma^{l_i}(\|l_i\|) + \gamma^{\tilde{u}_i}(\|\tilde{u}_i\|),\end{aligned}$$

as $t \to \infty$. Finally, one has that

$$\begin{aligned}\|E_i(t)\| \le{}& \frac{\zeta_M}{\sqrt{b_{iM}}} + \frac{(\sqrt{\gamma_{zi2}+1}\epsilon_M)}{\sqrt{b_{iM}}} + \frac{\sqrt{k_{i3}}\omega_M}{\sqrt{b_{iM}}} \\ &+ \frac{\sqrt{\gamma_{zi2}}b_i}{\sqrt{b_{iM}}} + \frac{l_M}{\sqrt{b_{iM}}},\end{aligned}$$

with $\lim_{t\to\infty} l_i \le l_M$ and $\lim_{t\to\infty} \tilde{u}_i = 0$ when $t \to \infty$.

Accordingly, the compensated signal errors ϖ_{i1}, ϖ_{i2}, the estimation error $\mathcal{Z}_i$ and the parameter error $\tilde{\omega}_i$ are ultimately bounded. From the boundness of the compensated signal z_{i1} and z_{i2} [31], one has that the fencing errors $e_{i1}, e_{i2}, i \in \mathcal{F}$, are bounded as well. The proof is thus completed. □

Remark 3.8. While the position-only design is more complex compared to the simple state feedback design, it offers the advantage of lower sensor costs, such as for velocity and acceleration sensors. As a result, this type of algorithm has greater potential for application in large-scale systems.

Remark 3.9. (i) To satisfy the input constraints of follower i, an optimal signal u_i^o via neurodynamic optimization is given $\underline{u}_i \le u_i^o \le \bar{u}_i$, with $\underline{u}_i, \bar{u}_i \in \mathbb{R}$ satisfying $u_{\min} = \underline{u}_i, \bar{u}_i = u_{\max}$. Then, we define a cost function,

$$\begin{aligned}&\min J_i(u_i^o) = \left\{\frac{\lambda_{i1}}{2}(u_i^o - u_i^c)^2 + \frac{\lambda_{i2}}{2}(u_i - u_i^o)^2\right\} \\ &\text{s.t. } h(u_i^o) \le 0, \quad i = 1, 2, \dots, n, \qquad (29)\end{aligned}$$

where $\lambda_{i1}, \lambda_{i2} \in \mathbb{R}^+$ are the optimal weights and $h(u_i^o) = \{-u_i^o + \underline{u}_i \le 0; u_i^o - \bar{u}_i \le 0\}$. (ii) To improve the calculation efficiency, a one-layer recurrent neural network (RNN) with finite-time convergence [34] is utilized, namely,

$$\epsilon\frac{du_i^o}{dt} = -[\nabla J_i(u_i^o) + \varsigma\nabla h(u_i^o)k_{[a,b]}(h(u_i^o))],$$

where $\epsilon \in \mathbb{R}^+$ is the time constant and $\varsigma \in \mathbb{R}^+$. $\nabla J_i(u_i^o)$ and $\nabla h(u_i^o)$ represent the gradient of $J_i(u_i^o)$ and $h(u_i^o)$, respectively. $k_{[a,b]}(s)$ is a limit function as

$$k_{[a,b]}(s) = \begin{cases} b, & s > b, \\ s, & a \le s \le b, \\ a, & s < a, \end{cases} \qquad (30)$$

where $a \in \mathbb{R}, b \in \mathbb{R}$ satisfy $b > a$ are the specified values.

Remark 3.10. The NN optimization utilizes only information of follower $i, i \in \mathcal{F}$, which implies that the fencing controller in (22) is executed in a distributed manner.

4. Numerical Simulation

In this section, we consider a target-follower MAS consisting of 7 followers and 3 targets. The topology of the MAS is illustrated in Fig. 1 under Assumption 2.2, where the connection among the follower set $\mathcal{F}$ is disconnected.

For the target set $\mathcal{T}$, the initial positions of targets are set to be $r_8(0) = [0, 25]^\top$m, $r_9(0) = [5, 15]^\top$m, $r_{10}(0) = [-10, 0]^\top$m with a common velocity $v_d = [0.7, 0.7]^\top$m/s in (2), which thus can maintain a convex hull $Co(\mathcal{T})$. To get the center of motional target set $\mathcal{T}$ with $G(\nu, \varepsilon)$, the parameters k, α for the observer $\hat{r}_i(t), \hat{v}_d^i(t), i \in F$ in (4) are set to be $k = 1, \alpha = 0.5$.

For the follower group $\mathcal{F}$, it follows from Assumptions 3.4 and 3.5 that the nonlinear terms are designed to be

$$f_1(\xi_1) = -0.5\sin(v_1^2) + 2.45\cos(v_1^2) + 0.1\sin(v_1)v_1,$$

$$f_2(\xi_2) = -0.5\sin(v_2^2) + 1.75\cos(v_2^2) + 0.1\sin(v_2)v_2,$$

$$f_3(\xi_3) = \sin(v_3^2) + 1.75\cos(v_3^2) + 0.2\sin(v_3)v_3,$$

$$f_4(\xi_4) = 0.6\sin(v_4^2) + 1.75\cos(v_4^2) + 0.4\sin(v_4)v_4,$$

$$f_5(\xi_5) = -\sin(v_5^2) + 2.8\cos(v_5^2) + 0.6\sin(v_5)v_5,$$

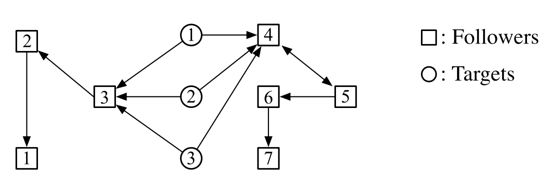

Fig. 1. Communication topology of the target-follower group system.

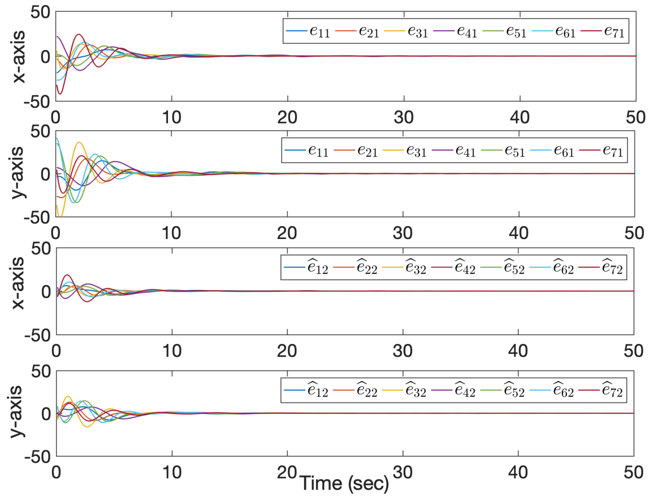

Fig. 2. The evolutions of the fencing position errors $e_{i1}, i \in \mathcal{F}$ in (10) and the modified fencing velocity errors $\hat{e}_{i2}, i \in \mathcal{F}$, in (20), respectively.

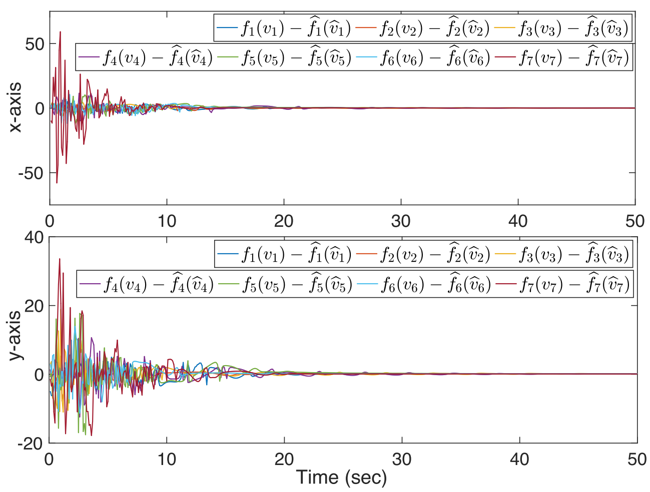

Fig. 3. The evolutions of the estimated errors of the nonlinear term $f_i(v_i) - \hat{f}_i(\hat{v}_i)$ with output feedback controller (22).

$$f_6(\xi_6) = -\sin(v_6^2) + 2.75\cos(v_6^2) + 0.8\sin(v_6)v_6,$$

$$f_7(\xi_7) = 0.6\sin(v_7^2) - 3.15\cos(v_7^2) + 0.3\sin(v_7)v_7.$$

For the proposed output feedback controller (22), the parameters are set to be $k_{i1} = 4, k_{i2} = 2, k_{i3} = 0.02, \gamma_i = 1, c_{i1} = 2, c_{i2} = 1, i \in \mathcal{F}$ according to Lemma 3.6. For the RBF and NNs design, the number of NN nodes is $N_i = 101, i \in \mathcal{F}$ for all $f_i(v_i)$, and the output $\theta_i(v_i)$ of the RBF node is chosen to be $\exp(-\|v_i - c_i\|^2/2b_i^2)$, where the parameters $c_i, i = 1-7$ are essentially designed to be evenly spaced in the region of [5; 5]; [5; 5]; [15; 15]; [5; 5]; [10; 10]; [10; 10]; [25; 25], respectively, and $b_1 = 0.38, b_2 = 0.38, b_3 = 0.3, b_4 = 0.45, b_5 = 0.25, b_6 = 0.75, b_7 = 0.72$. Moreover, due to all the positions of the targets being available to the well-informed followers, the fencing radius is chosen as $\rho = 30$ m in Definition 2.4.

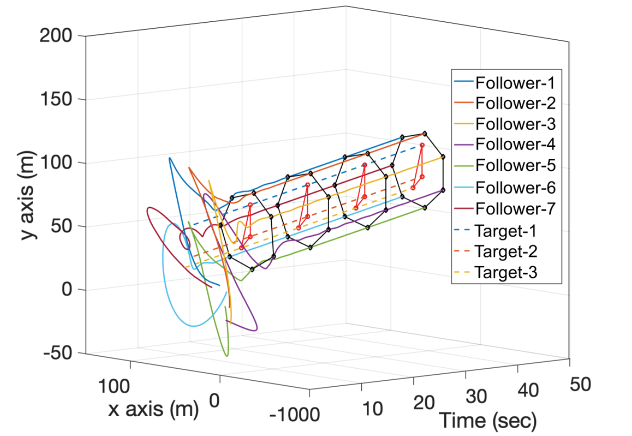

Fig. 4. Trajectories of seven robots using the proposed fencing controller (22) from initial positions to the final equal-fencing formation for the moving target group. Here, the black rectangles represent the follower group with solid trajectories; the red circles represent the target group with dashed trajectories.

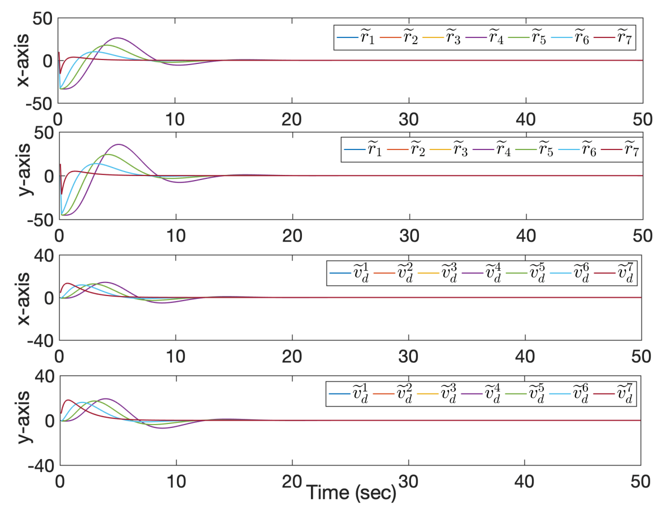

Fig. 5. The evolution of the estimated position error $\tilde{r}_i, \in \mathcal{F}$, and the velocity errors $\tilde{v}_d^i, \in \mathcal{F}$, for the distributed estimator of the target's center in (4).

Precisely, as shown in Fig. 4, seven followers (black rectangles) from different initial positions achieve a regular polygon formation for the three targets of the constant velocity (red circles). In what follows, we illustrate the detailed evolution of states in the equal-fencing formation. Precisely, it is observed in Fig. 5 that the evolutions of estimated position errors $\tilde{r}_i$, $i \in \mathcal{F}$, and position errors $\tilde{v}_d^i$, $i \in \mathcal{F}$, in Eq. (6), respectively, drop in less than 16 s, i.e. $\lim_{t\to 16s} \tilde{r}_i(t) = 0, \lim_{t\to 16s} \tilde{v}_d^i(t) = 0$, which vividly verifies the feasibility of Lemma 3.1. As shown in Fig. 6, one has that the estimated position and velocity errors $\tilde{x}_i, \tilde{v}_i, i \in \mathcal{F}$, in (18) for the positions and velocities x_i, v_i of the follower $i, i \in \mathcal{F}$, converge to

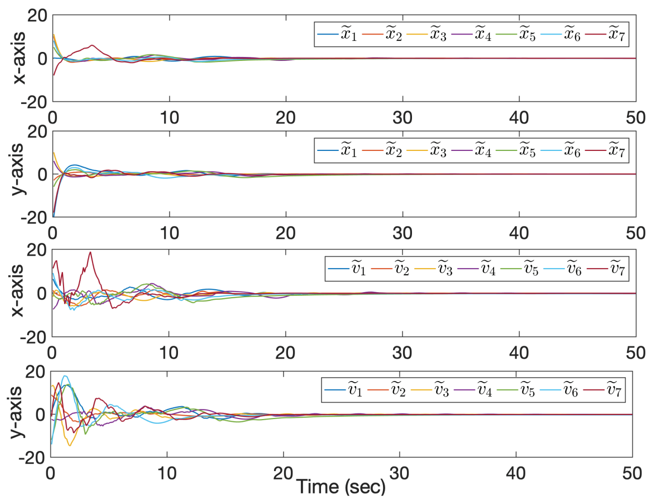

Fig. 6. The evolutions of the estimated position and velocity errors $\tilde{x}_i, \tilde{v}_i, i \in \mathcal{F}$, in (18) for the positions and velocities x_i, v_i of the follower $i, i \in \mathcal{F}$.

zeros, i.e. $\lim_{t\to\infty} \tilde{x}_i(t) = 0, \lim_{t\to\infty} \tilde{v}_i(t) = 0$, which demonstrates the effectiveness of the output feedback design. Moreover, it is observed in Fig. 2 that the fencing position errors $e_{i1}, i \in \mathcal{F}$ and modified velocity errors $\hat{e}_{i2}, i \in \mathcal{F}$, settle to zeros 19 s, i.e. $\lim_{t\to 19s} e_{i1}(t) = 0, \lim_{t\to 19s} \hat{e}_{i2}(t) = 0$, which verifies that the fencing formation is achieved. Additionally, the difference of nonlinear term estimation $f_i(v_i) - \hat{f}_i(\hat{v}_i)$ are depicted in Fig. 3, which settles to zero in less than 28 s, i.e. $\lim_{t\to 28s} f_i(v_i(t)) - \hat{f}_i(\hat{v}_i(t)) = 0$. Therefore, the convergence of the equal fencing with output feedback controller (22) is guaranteed, which has more inspiring potential in real applications. However, it may pose more constraints on the nonlinear term $f_i(v_i)$ to achieve the fencing objectives.

5. Conclusion

In this paper, we have proposed distributed fencing control protocols for constrained networked second-order nonlinear MASs with output feedback. With such a fencing control protocol, the follower group encircles multiple motional targets with evenly distributed phases around the circle. Utilizing the output feedback laws, the fencing protocol has achieved equal fencing via only position information. The conditions are derived to guarantee the stability of the proposed control protocol as well. Numerical simulations are conducted to show the effectiveness of the proposed methods. Such a fencing control method has the potential for hunting, patrolling resources, aquatic resource explorations, rescue, supplies delivery, and escort of abundant industrial multi-unmanned system swarms. Future work will explore multi-robot fencing control for multiple moving targets with unknown and limited velocities, time-varying communication topologies, and even in environments with obstacles.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grants 62225306 and U2141235, in part by the National Key Research and Development Program of China under Grant 2022ZD0119601, and in part by Guangdong Basic and Applied Research Foundation under Grant 2022B1515120069.

ORCID

Bin-Bin Hu
https://orcid.org/0000-0003-3157-4309
Hai-Tao Zhang
https://orcid.org/0000-0002-8819-8829

References

[1] H. Wei, C. Liu and Y. Shi, A robust distributed MPC framework for multi-agent consensus with communication delays, *IEEE Trans. Autom. Control* **69**(11) (2024) 1–15, doi:10.1109/TAC.2024.3389075.
[2] B.-B. Hu, H.-T. Zhang, W. Yao, J. Ding and M. Cao, Spontaneous-ordering platoon control for multirobot path navigation using guiding vector fields, *IEEE Trans. Robot.* **39**(4) (2023) 2654–2668.
[3] B.-B. Hu, H.-T. Zhang, W. Yao, Z. Sun and M. Cao, Coordinated guiding vec-
tor field design for ordering-flexible multi-robot surface navigation, *IEEE Trans. Autom. Control* **69**(7) (2024) 1–8, doi:10.1109/TAC.2024.3355321.
[4] H. Su, G. Chen, X. Wang and Z. Lin, Adaptive second-order consensus of networked mobile agents with nonlinear dynamics, *Automatica* **47**(2) (2011) 368–375.
[5] J. Shao, W. X. Zheng, T.-Z. Huang and N. A. Bishop, On leader–follower consensus with switching topologies: An analysis inspired by pigeon hierarchies, *IEEE Trans. Autom. Control* **63**(10) (2018) 3588–3593.
[6] P. Li, F. Jabbari and X.-M. Sun, Containment control of multi-agent systems with input saturation and unknown leader inputs, *Automatica* **130** (2021) 109677.
[7] B.-B. Hu and H.-T. Zhang, Bearing-only motional target-surrounding control for multiple unmanned surface vessels, *IEEE Trans. Ind. Electron.* **69**(4) (2021) 3988–3997.
[8] B.-B. Hu, Y. Zhou, H. Wei, Y. Wang and C. Lv, Ordering-flexible multi-robot coordination for moving target convoying using long-term task execution, *Automatica* **163** (2024) 111558.
[9] B.-B. Hu, H.-T. Zhang and Y. Shi, Cooperative label-free moving target fencing for second-order multi-agent systems with rigid formation, *Automatica* **148** (2023) 110788.
[10] B.-B. Hu, H.-T. Zhang, B. Liu, J. Ding, Y. Xu, C. Luo and H. Cao, Coordinated navigation control of cross-domain unmanned systems via guiding vector fields, *IEEE Trans. Control Syst. Technol.* **32**(2) (2023) 550–563.
[11] C. Wang and G. Xie, Limit-cycle-based decoupled design of circle formation control with collision avoidance for anonymous agents in a plane, *IEEE Trans. Autom. Control* **62**(12) (2017) 6560–6567.
[12] Z. Chen, A cooperative target-fencing protocol of multiple vehicles, *Automatica* **107** (2019) 591–594.
[13] X. Yu and L. Liu, Distributed circular formation control of ring-networked nonholonomic vehicles, *Automatica* **68** (2016) 92–99.

[14] X. Yu and L. Liu, Cooperative control for moving-target circular formation of nonholonomic vehicles, *IEEE Trans. Autom. Control* **62**(7) (2016) 3448–3454.

[15] F. Chen, W. Ren and Y. Cao, Surrounding control in cooperative agent networks, *Syst. Control Lett.* **59**(11) (2010) 704–712.

[16] S. Shoja, M. Baradarannia, F. Hashemzadeh, M. Badamchizadeh and P. Bagheri, Surrounding control of nonlinear multi-agent systems with non-identical agents, *Syst. Control Lett.* **70** (2017) 219–227.

[17] Y. Shi, R. Li, Rui and L. K. Teo, Cooperative enclosing control for multiple moving targets by a group of agents, *Int. J. Control* **88**(1) (2015) 80–89.

[18] R. Li, Y. Shi and Y. Song, Multi-group coordination control of multi-agent system based on smoothing estimator, *IET Control Theory Appl.* **10**(11) (2016) 1224–1230.

[19] X. Dong and G. Hu, Time-varying formation tracking for linear multiagent systems with multiple leaders, *IEEE Trans. Autom. Control* **62**(7) (2017) 3658–3664.

[20] J. Yu, X. Dong, Z. Liang, Q. Li and Z. Ren, Practical time-varying formation tracking for high-order nonlinear multiagent systems with multiple leaders based on the distributed disturbance observer, *Int. J. Robust Nonlinear Control* **28**(9) (2018) 3258–3272.

[21] B.-B. Hu, H.-T. Zhang and J. Wang, Multiple-target surrounding and collision avoidance with second-order nonlinear multiagent systems, *IEEE Trans. Ind. Electron.* **68**(8) (2020) 7454–7463.

[22] B.-B. Hu, W. Yao, H. Wei and C. Lv, FSOC: Flexible subgrouping and ordering of multi-robot coordination for convoying multiple scattered targets, *IEEE Trans. Control Netw. Syst.* (2024) 1–12, doi:10.1109/TCNS.2024.3393643.

[23] X. Wang, H. Su, X. Wang and G. Chen, Fully distributed event-triggered semiglobal consensus of multi-agent systems with input saturation, *IEEE Trans. Ind. Electron.* **64**(6) (2016) 5055–5064.

[24] T. Yang, Z. Meng, V. D. V. Dimarogonas and H. K. Johansson, Global consensus for discrete-time multi-agent systems with input saturation constraints, *Automatica* **50**(2) (2014) 499–506.

[25] H.-T. Zhang, Z. Cheng, G. Chen and C. Li, Model predictive flocking control for second-order multi-agent systems with input constraints, *IEEE Trans. Circuits Syst. I, Reg. Papers* **62**(6) (2015) 1599–1606.

[26] Z. Peng, J. Wang and Q.-L. Han, Path-following control of autonomous underwater vehicles subject to velocity and input constraints via neurodynamic optimization, *IEEE Trans. Ind. Electron.* **66**(11) (2018) 8724–8732.

[27] Y. Wang, Z. Zhang, H. Wei, G. Yin, H. Huang, B. Li and C. Huang, A novel fault-tolerant scheme for multi-model ensemble estimation of tire road friction coefficient with missing measurements, *IEEE Trans. Intell. Veh.* **9**(1) (2023) 1066–1078.

[28] Y. Wang, F. Hu, C. Tian, P. Li, H. Huang, G. Yin and C. Huang, FTEKFNet: Hybridizing physical and data-driven estimation algorithms for vehicle state, *IEEE Trans. Intell. Veh.* **9**(11) (2024) 1–11, doi:10.1109/TIV.2024.3395911.

[29] B.-B. Hu, H.-T. Zhang, B. Liu, H. Meng and G. Chen, Distributed surrounding control of multiple unmanned surface vessels with varying interconnection topologies, *IEEE Trans. Control Syst. Technol.* **30**(1) (2021) 400–407.

[30] R. M. Sanner and J.-J. E. Slotine, Gaussian networks for direct adaptive control, *IEEE Trans. Neural Netw.* **3**(6) (1992) 837–863.

[31] B. Xu, Z. Shi, C. Yang and F. Sun, Composite neural dynamic surface control of a class of uncertain nonlinear systems in strict-feedback form, *IEEE Trans. Cybern.* **44**(12) (2014) 2626–2634.

[32] S. K. Narendra and K. Parthasarathy, Identification and control of dynamical systems using neural networks, *IEEE Trans. Neural Netw.* **1**(1) (1990) 4–27.

[33] C. Wang, J. D. Hill, S. S. Ge and G. Chen, An ISS-modular approach for adaptive neural control of pure-feedback systems, *Automatica* **42**(5) (2006) 723–731.

[34] Q. Liu and J. Wang, A one-layer recurrent neural network for constrained nonsmooth optimization, *IEEE Trans. Syst. Man Cybern. B Cybern.* **41**(5) (2011) 1323–1333.

Bin-Bin Hu is a Postdoc Researcher at the ENTEG, Faculty of Science and Engineering, University of Groningen, 9747 AG Groningen, The Netherlands. He obtained a Ph.D. degree in control science and engineering from Huazhong University of Science and Technology, Wuhan, China, in 2022. From August 2022 to August 2024, he was a Research Fellow with the School of Mechanical and Aerospace Engineering at Nanyang Technological University, Singapore. He was the recipient of the excellent doctoral dissertation award of the Chinese Association of Automation in 2023. His research interests include control of networked systems, multi-robot systems, and autonomous navigation.

Hai-Tao Zhang is a Professor at the School of Artificial Intelligence and Automation, the Engineering Research Center of Autonomous Intelligent Unmanned Systems, the Key Laboratory of Image Processing and Intelligent Control, and the State Key Lab of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan 430074, P.R. China. He received his B.E. and Ph.D. degrees in automation and control science and technology from the University of Science and Technology of China, Hefei, China, in 2000 and 2005, respectively. In 2007, he was a Postdoctoral Researcher with the University of Cambridge, Cambridge, U.K. Since 2005, he has been with the Huazhong University of Science and Technology, Wuhan, China, where he was an Associate Professor from 2005 to 2010, and has been a Full Professor since 2010. His research interests include swarming intelligence, model predictive control, and unmanned system cooperation control. Dr. Zhang was a recipient of the National Science Fund for Distinguished Young Scholars. He is/was an Associate Editor for IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS: SYSTEMS, IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS-PART II: EXPRESS BRIEFS, Engineering, Unmanned Systems, Asian Journal of Control, IEEE Conference on Decision and Control, and American Control Conference.