

## Topology-Aware Data-Driven Event-Triggered Control: An Optimization-Based Approach

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This paper presents a topology-aware framework for data-driven event-triggered control of unknown continuous-time linear systems. Addressing the conservatism of conventional isotropic triggering conditions, we propose a dual utilization paradigm where offline data are used to synthesize both the feedback controller and a structure-embedded triggering mechanism. We introduce a generalized triggering structure that reshapes the safety region to align with the system's learned energy landscape. An optimization-based synthesis is formulated to maximize the safety region while strictly guaranteeing global exponential stability and a strictly positive Minimum Inter-Event Time (MIET). Numerical experiments validate that the proposed strategy significantly reduces communication frequency compared to the isotropic baselines.

**Keywords:** Data-driven control; event-triggered control; topology-aware design; linear matrix inequalities (LMI).

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### 1. Introduction

With the rapid evolution of communication and computing technologies, networked control systems and cyber-physical systems have become ubiquitous in modern engineering. In these environments, efficient utilization of shared resources is paramount, rendering traditional periodic sampling increasingly inefficient. To achieve efficient utilization of shared resources, *event-triggered control* has

emerged as a pivotal resource-aware paradigm [1, 2]. Unlike time-triggered schemes, event-triggered control transmits updates only when specific state-dependent conditions are violated. This on-demand execution significantly reduces network traffic while preserving stability, leading to various frameworks including static relative thresholding [1, 3] and dynamic triggering mechanisms [4, 5].

Event-triggered control has been substantially enriched to address practical constraints. Heemels *et al.* proposed periodic event-triggered control for digital platforms where continuous monitoring is infeasible [6]. Donkers and Heemels extended the framework to observer-based output-feedback control [7]. To further reduce communication, Girard developed dynamic triggering mechanisms to filter out Zeno behaviors [4]. The paradigm has also been unified for nonlinear plants [5], scaled to distributed multi-agent

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coordination [8], and integrated with model predictive control [9]. However, a pervasive assumption in these strategies is the availability of an accurate mathematical model. In many scenarios, obtaining precise models is prohibitively expensive or impossible due to increasing system complexity.

Because of its ability to circumvent the limitations of the model-based strategies, *direct data-driven control* has witnessed a renaissance. The cornerstone of this resurgence is Willems' fundamental lemma [10], which states that a single rich trajectory can characterize a linear system's behavior. Leveraging this result, Markovsky and Rapisarda laid the groundwork for data-driven simulation and control [11]. More recently, De Persis and Tesi derived explicit formulas to synthesize stabilizing controllers directly from data by solving linear matrix inequalities [12]. In optimal control, Coulson *et al.* introduced data-enabled predictive control [13]. This field has been further reinforced by analyses of data informativity [14] and robust design techniques for overcoming measurement noise [15, 16]. Furthermore, the framework has been extended to nonlinear systems via approximate nonlinearity cancellation [17] or Koopman operator theory [18].

Integrating these data-driven techniques with resource-aware mechanisms has given rise to *data-driven event-triggered control*. Initial efforts focused on discrete-time systems, with Wang *et al.* designing robust controllers under bounded noise [19] and Digge *et al.* exploring noise-free settings [20]. This line of research has extended to distributed network systems [21] and alternative modeling approaches like fuzzy granular models [22]. To reduce online monitoring, Liu *et al.* proposed a data-driven self-triggered strategy based on trajectory prediction [23].

For continuous-time systems, Wei *et al.* combined periodic sampling with event-triggered updates [24], and Qi *et al.* extended the framework to guarantee  $\mathcal{L}_2$ -gain performance [25]. Most recently, De Persis *et al.* established a robust framework with explicit minimum inter-event times via static thresholding [26]. Concurrently, Xu *et al.* introduced a dynamic triggering mechanism to ensure exponential stability [27]. Furthermore, Feng *et al.* proposed a co-design approach optimizing the controller and triggering condition using imperfect data [28].

Despite these developments, a common limitation persists: existing triggering conditions are predominantly *isotropic*. For instance, the weighting matrices in [26–28] rely on scaled identity matrices, imposing uniform spherical constraints on the error. Such conditions fail to exploit the directional structural information of the system's energy landscape, often leading to conservative sampling in the robustness directions.

To address this conservatism, this paper proposes a novel *topology-aware data-driven event-triggered control*

framework. Before detailing the contributions, it is crucial to clarify the terminology: in this context, "*topology*" does not refer to the communication graph structure commonly seen in multi-agent systems. Instead, it refers to the deeper geometric properties of the system's state space, specifically, the topology of the *Lyapunov level sets* (energy landscape) and the vector field geometry. By extracting the intrinsic Lyapunov matrix and the dissipative dynamics directly from offline data, our proposed controller effectively "shapes" the triggering manifold, ensuring that it aligns seamlessly with the natural physical characteristics of the plant rather than imposing an arbitrary spherical constraint.

The main contributions of this work are summarized as follows:

- (1) *Topology-aware triggering via dual data utilization*: We propose a novel design paradigm where offline data are utilized not only to synthesize the feedback controller but also to construct the event-triggering mechanism. By directly embedding the learned Lyapunov energy metric and the coupling dynamics into the triggering logic, we introduce a generalized condition decomposed into state threshold scaling, error sensitivity weighting, and dissipative coupling. This structural innovation transforms the safety region from a rigid sphere to a flexible hyper-ellipsoid, enabling the triggering manifold to align seamlessly with the system's natural energy landscape. Note that the safety region is defined as the subset of the state space where no event is triggered.
- (2) *Optimization-based synthesis with guarantees*: We formulate a convex optimization problem to optimally shape the triggering manifold based on the learned system topology. By maximizing the state energy threshold, the proposed strategy significantly reduces communication frequency compared to static isotropic methods. Furthermore, the framework rigorously guarantees exponential stability and the existence of a strictly positive minimum inter-event time for the closed-loop system.

The remainder of this paper is organized as follows. Section 2 introduces some preliminaries on system dynamics and its data-driven representation, reviews the conventional static isotropic strategy, and formulates the data-driven control problem. Section 3 details the proposed topology-aware framework, covering structural decomposition, optimization synthesis, and theoretical analysis. Section 4 presents numerical validation, followed by conclusions in Sec. 5.

### 1.1. Notation

$\mathbb{R}$ ,  $\mathbb{R}_{\geq 0}$ , and  $\mathbb{N}_0$  denote the sets of real numbers, non-negative real numbers, and non-negative integers, respectively.  $\mathbb{R}^{n \times m}$  represents the space of  $n \times m$  real matrices.  $\mathbb{S}^n$  denotes

the set of  $n \times n$  symmetric matrices. For a matrix  $M$ ,  $M^T$  denotes its transpose, and  $\|M\|$  denotes its spectral norm (induced 2-norm). The notation  $M > 0$  ( $M \geq 0$ ) means that  $M$  is symmetric positive definite (positive semi-definite). In symmetric block matrices, the symbol  $\star$  represents the elements induced by symmetry.  $I$  and  $0$  denote the identity matrix and zero matrix of appropriate dimensions. For a vector  $x$ ,  $\|x\|$  denotes the Euclidean norm.  $\lambda_{\min}(M)$  and  $\lambda_{\max}(M)$  denote the minimum and maximum eigenvalues of a symmetric matrix  $M$ , respectively.

## 2. Preliminaries and Problem Formulation

### 2.1. System dynamics and data-driven representation

Consider a continuous-time linear time-invariant system,

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  is the state and  $u(t) \in \mathbb{R}^m$  is the control input. System matrices  $A$  and  $B$  are constant but *unknown*. We assume that the pair  $(A, B)$  is stabilizable, which is a necessary condition for the existence of a stabilizing controller.

We consider a networked control scenario where the control input is updated only at discrete transmission instants  $\{t_k\}_{k \in \mathbb{N}_0}$ , with  $t_0 = 0$ . Between two consecutive transmission instants, the control input is held constant by a Zero-Order Hold (ZOH) device,

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1}), \quad (2)$$

where  $K \in \mathbb{R}^{m \times n}$  is a state-feedback gain to be designed.

We define the sampling-induced error  $e(t)$  as

$$e(t) := x(t_k) - x(t), \quad t \in [t_k, t_{k+1}). \quad (3)$$

Then, the closed-loop system can be written as

$$\dot{x}(t) = (A + BK)x(t) + BKe(t). \quad (4)$$

Since  $A$  and  $B$  are unknown, we rely on data collected offline to design the controller and the triggering mechanism. We assume an experiment is conducted on the system over a time interval  $[0, T_{\text{end}}]$ , with state and input measurements recorded with a sampling period  $T_s$ . The collected data are organized into the following Hankel-like matrices

$$\begin{aligned} U_0 &:= [u(0) \quad u(T_s) \quad \cdots \quad u((T-1)T_s)], \\ X_0 &:= [x(0) \quad x(T_s) \quad \cdots \quad x((T-1)T_s)], \\ X_1 &:= [\dot{x}(0) \quad \dot{x}(T_s) \quad \cdots \quad \dot{x}((T-1)T_s)], \end{aligned}$$

where  $T$  is the number of samples. Note that  $X_1$  contains the state derivatives, which are assumed to be available

or accurately approximated from the measurements. We assume a condition on the richness of the data.

**Assumption 2.1.** The matrix  $[U_0^T \quad X_0^T]^T$  is of full row rank.

Under Assumption 2.1, the fundamental lemma [10] guarantees that the subspace of input-state trajectories is fully captured by the data. Specifically, for any given state-feedback gain  $K \in \mathbb{R}^{m \times n}$ , there exists a matrix  $G \in \mathbb{R}^{T \times n}$  satisfying the data equation,

$$\begin{bmatrix} K \\ I_n \end{bmatrix} = \begin{bmatrix} U_0 \\ X_0 \end{bmatrix} G.$$

Consequently, the closed-loop system matrix can be exactly represented as  $A + BK = X_1 G$  [12]. Similarly, to isolate the input term, there exists a matrix  $L \in \mathbb{R}^{T \times n}$  satisfying

$$\begin{bmatrix} K \\ 0 \end{bmatrix} = \begin{bmatrix} U_0 \\ X_0 \end{bmatrix} L,$$

which implies  $BK = X_1 L$ . These data-driven representations ( $X_1 G$  and  $X_1 L$ ) allow us to analyze the closed-loop dynamics (4) and design the triggering mechanism without explicit knowledge of  $A$  and  $B$ .

**Remark 2.2.** Note that the construction of  $X_1$  involves state derivatives. In practice, the state derivatives can be approximated using numerical differentiation techniques, as extensively discussed in [26]. Furthermore, alternative data acquisition strategies considering signal quantization effects have been investigated in [27].

### 2.2. Static isotropic event-triggering

To facilitate the subsequent design, we briefly review the static isotropic event-triggered control framework [26]. This approach typically employs a static triggering condition of the form  $\|e(t)\| \leq \sigma \|x(t)\|$ , which imposes a spherical constraint on the sampling error. Then the triggering times are defined as follows:  $t_0 = 0$  and

$$t_{k+1} = \inf\{t > t_k : z(t)^T \Psi_{\text{static}}(\sigma) z(t) > 0\},$$

where

$$\begin{aligned} \Psi_{\text{static}}(\sigma) &= \text{diag}(-\sigma^2 I_n, I_n) \quad \text{and} \\ z(t) &= [x(t)^T e(t)^T]^T. \end{aligned}$$

First, a data-driven stabilizing state-feedback gain  $K$  is designed such that  $A + BK$  is Hurwitz. Following [12], if there exists a matrix  $Y$  satisfying  $X_1 Y + (X_1 Y)^T < 0$  and  $X_0 Y > 0$ , the controller is explicitly given by  $K = U_0 Y (X_0 Y)^{-1}$ . Furthermore, matrix  $S := (X_0 Y)^{-1}$  defines a Lyapunov function  $V(x) = x^T S x$ . Substituting the derived data-based dynamics  $\dot{x}(t) = X_1 G x(t) + X_1 L e(t)$  into the time derivative of the

Lyapunov function  $V(x) = x^T S x$ , we obtain

$$\begin{aligned} \dot{V}(x) &= \dot{x}(t)^T S x(t) + x(t)^T S \dot{x}(t) \\ &= (X_1 G x + X_1 L e)^T S x + x^T S (X_1 G x + X_1 L e) \\ &= \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}^T \underbrace{\begin{bmatrix} (S X_1 G)^T + S X_1 G & S X_1 L \\ (S X_1 L)^T & 0 \end{bmatrix}}_M \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}. \end{aligned} \quad (6)$$

Recalling that  $z(t) = [x(t)^T, e(t)^T]^T$ , we have  $\dot{V}(x) = z(t)^T M z(t)$ .

Consequently, the triggering parameter  $\sigma$  must satisfy the stability condition  $\mu M - \Psi_{\text{static}}(\sigma) < 0$  for some  $\mu > 0$ . A crucial property of this static strategy is the existence of a guaranteed Minimum Inter-Event Time (MIET), which ensures the exclusion of Zeno behavior. As derived in [26], the inter-event times are globally lower-bounded by

$$\tau(\sigma) := \frac{1}{\Gamma} \frac{\sigma}{1 + \sigma}, \quad \Gamma := \max\{\|X_1 G\|, \|X_1 L\|\}. \quad (7)$$

Despite these theoretical guarantees, the static mechanism imposes a fixed spherical constraint on the error, which implicitly assumes that the system's sensitivity to error is uniform in all directions. Such an assumption ignores the directional structural information of the system dynamics (e.g. the shape of the Lyapunov level sets), leading to conservative sampling behavior. This limitation motivates the topology-aware mechanism we are to propose in this paper.

**Problem Statement:** Based on the motivation discussed above, the objective of this work is to design a topology-aware event-triggering strategy strictly using the offline data  $\{U_0, X_0, X_1\}$ . Specifically, the proposed mechanism is required to guarantee the exponential stability of the closed-loop system while simultaneously ensuring a strictly positive MIET to exclude Zeno behavior.

### 3. Main Results

In this section, we present the proposed topology-aware event-triggered control framework. Having established the stabilizing controller  $K$  using the standard data-driven approach outlined in Sec. 2, our focus now shifts to the design of the triggering mechanism itself.

#### 3.1. Overview of the design framework

The overall architecture of the proposed strategy is illustrated in Fig. 1. The framework operates by bridging two distinct phases: offline data learning and online event-triggered execution.

The distinctive feature of our approach, as highlighted in the schematic, is the dual utilization of the offline dataset

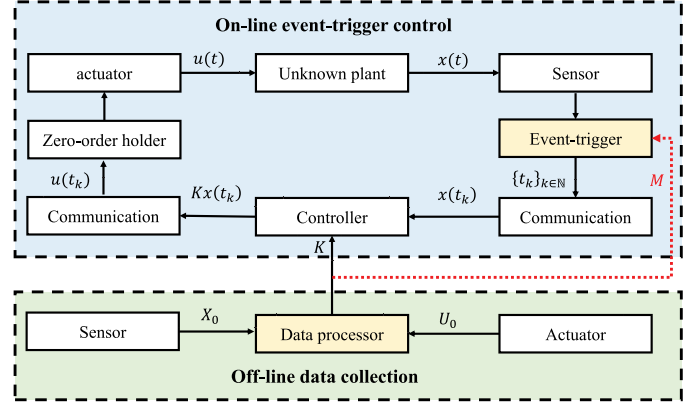


Fig. 1. Schematic diagram of the proposed topology-aware data-driven event-triggered control framework. The framework consists of two distinct phases: (i) Off-line Data Collection (Bottom): Input-state trajectories ( $U_0, X_0$ ) are collected from the unknown plant to capture the system dynamics. (ii) On-line Event-Triggered Control (Top): The collected data are utilized to synthesize both the feedback controller  $K$  and the topology-aware triggering mechanism, which regulates the data transmission over the network.

$\{U_0, X_0\}$ . In traditional data-driven control paradigms, historical data are primarily consumed to synthesize the control law  $u(t) = Kx(t_k)$ . However, this leaves the event-triggering mechanism reliant on generic, isotropic thresholds that ignore the system's specific dynamical structure.

To overcome this limitation, our framework introduces a novel pathway shown in Fig. 1, where the *Data Processor* extracts structural information from the offline data and injects it directly into the *Event-Trigger* block. Specifically, the collected input-state trajectories are used to reconstruct the underlying topology of the closed-loop energy landscape. This learned topology is then embedded into the online triggering logic. Consequently, the decision to trigger a transmission is no longer based solely on the magnitude of the sampling error  $e(t)$ , but on its alignment with the data-derived system structure.

By doing so, the event generator becomes “topology-aware”, capable of distinguishing between benign and critical sampling errors based on the geometric features learned from the offline experiment. In the following subsections, we mathematically formalize this structural decomposition and derive the optimization-based synthesis procedure.

#### 3.2. Physics-informed structural decomposition

To address the conservatism of static isotropic triggering, we propose a generalized topology-aware triggering structure. Leveraging the optimal isotropic parameter  $\sigma^*$  as a reference, we introduce a structural relaxation that transforms the rigid spherical constraint into a flexible geometric region adapted to the system's energy landscape.

Let  $S$  be the Lyapunov matrix and  $K$  be the controller gain derived from the offline controller synthesis. We first

define the data-driven coupling matrix  $\Phi \in \mathbb{R}^{n \times n}$  as

$$\Phi = SX_1L. \quad (8)$$

Based on these structural matrices, we construct the proposed *Generalized Topology-Aware* triggering matrix  $\bar{\Psi}$  as an explicit linear combination of three components,

$$\begin{aligned} \bar{\Psi}(\alpha_x, \alpha_e, \gamma) := & \alpha_x \begin{bmatrix} -\sigma_*^2 S & 0 \\ 0 & 0 \end{bmatrix} + \alpha_e \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix} \\ & + \gamma \begin{bmatrix} 0 & \Phi \\ \Phi^T & 0 \end{bmatrix}, \end{aligned} \quad (9)$$

where  $\alpha_x > 0$ ,  $\alpha_e > 0$ , and  $\gamma \in \mathbb{R}$  are decision variables. This decomposed form clearly isolates the contribution of each physical mechanism as discussed below.

*State Threshold Scaling* ( $\alpha_x$ ): The first term in (9) determines the energy-dependent threshold on the state. By incorporating the Lyapunov matrix  $S$ , the isotropic threshold  $-\sigma_*^2 I$  is replaced by  $-\sigma_*^2 S$ . The decision variable  $\alpha_x$  acts as a dilation factor. Maximizing  $\alpha_x$  effectively elevates the upper bound of the admissible error energy, thereby creating a larger safety region and reducing the triggering frequency.

*Error Sensitivity Weight* ( $\alpha_e$ ): The second term defines the penalty on the sampling error. Unlike the static isotropic strategy which uses the identity matrix, this term weighs the error according to the system's energy metric  $S$ . Decoupling  $\alpha_e$  from  $\alpha_x$  provides the optimization algorithm with the flexibility to adjust the aspect ratio of the triggering ellipsoid. This allows the mechanism to be less sensitive in directions where the system is naturally robust.

*Dissipative Coupling* ( $\gamma$ ): The third term introduces a directional coupling between the state  $x(t)$  and the error  $e(t)$  via the matrix  $\Phi$ . Since the time derivative of the Lyapunov function contains the cross-term  $2x^T SX_1 L e$ , the parameter  $\gamma$  allows the triggering condition to exploit the instantaneous interaction between the error and the system dynamics. Specifically, it enables the mechanism to distinguish between "harmful" errors (which increase  $\dot{V}$ ) and "beneficial" errors (which contribute to energy dissipation). By tuning  $\gamma$ , the trigger can delay sampling when the error vector is aligned beneficially with the system dynamics, a feature completely absent in decoupled designs.

### 3.3. Optimization-based synthesis

Having defined the generalized structural form of  $\bar{\Psi}(\alpha_x, \alpha_e, \gamma)$ , we now proceed to synthesize the optimal parameters. The design goal is to maximize the safety region in the state space while rigorously guaranteeing two critical properties: (i) the exponential stability of the closed-loop system, and (ii) the existence of a strictly positive MIET to exclude Zeno behavior.

We formulate the synthesis problem as a convex optimization. As analyzed in Sec. 3.2, the parameter  $\alpha_x$  acts as the scaling factor for the state energy threshold. Maximizing  $\alpha_x$  directly elevates the upper bound of the admissible error, thereby extending the inter-event intervals. Therefore, we adopt the maximization of  $\alpha_x$  as the primary objective.

**Problem 3.1 (Topology-aware Synthesis).** Given the isotropic parameter  $\sigma_*$  and the Lyapunov matrix  $S$ , find the optimal scalars  $\alpha_x, \alpha_e, \gamma, \mu$  by solving the following Semidefinite Program (SDP):

$$\underset{\alpha_x, \alpha_e, \gamma, \mu}{\text{maximize}} \quad \alpha_x \quad (10a)$$

$$\text{subject to} \quad \mu M - \bar{\Psi}(\alpha_x, \alpha_e, \gamma) < 0, \quad (10b)$$

$$\bar{\Psi}(\alpha_x, \alpha_e, \gamma) \leq \Psi_{\text{static}}(\sigma_*), \quad (10c)$$

$$\alpha_x > 0, \quad \alpha_e > 0, \quad \mu > 0, \quad (10d)$$

where  $M$  is the data-based dissipation matrix defined in Eq. (6) and  $\Psi_{\text{static}}(\sigma_*) = \text{diag}(-\sigma_*^2 I_n, I_n)$  is the static isotropic triggering matrix.

The theoretical implications of the constraints are detailed as follows.

*Stability Guarantee* (10b): This strict Linear Matrix Inequality (LMI) ensures that the Lyapunov function strictly decreases along the trajectory. Specifically, it guarantees  $\dot{V}(z) < 0$  whenever the triggering condition  $z^T \bar{\Psi} z \leq 0$  is satisfied. The variable  $\mu$  acts as an S-procedure multiplier. This constraint implicitly requires the error sensitivity weight  $\alpha_e$  to be sufficiently positive to counteract the destabilizing effects of the coupling terms, ensuring the negative definiteness of the dissipation inequality.

*Zeno-freeness via Set Inclusion* (10c): This constraint is pivotal for preserving the MIET property. Let us explicitly define the safety regions (i.e. the sets of states where no transmission is triggered) for the static isotropic strategy and the proposed strategy as

$$\mathcal{S}_{\text{static}} := \{z \in \mathbb{R}^{2n} \mid z^T \Psi_{\text{static}}(\sigma_*) z \leq 0\},$$

$$\mathcal{S}_{\text{new}} := \{z \in \mathbb{R}^{2n} \mid z^T \bar{\Psi}(\alpha_x, \alpha_e, \gamma) z \leq 0\}.$$

The matrix inequality  $\bar{\Psi} \leq \Psi_{\text{static}}(\sigma_*)$  guarantees that for any  $z \in \mathbb{R}^{2n}$ , the scalar inequality  $z^T \bar{\Psi} z \leq z^T \Psi_{\text{static}} z$  holds. Consequently, we have the following implication:

$$\begin{aligned} z \in \mathcal{S}_{\text{static}} & \Rightarrow z^T \Psi_{\text{static}} z \leq 0 \\ & \Rightarrow z^T \bar{\Psi} z \leq 0 \Rightarrow z \in \mathcal{S}_{\text{new}}. \end{aligned}$$

This establishes the set inclusion  $\mathcal{S}_{\text{static}} \subseteq \mathcal{S}_{\text{new}}$ . Since the system trajectory must exit the larger set  $\mathcal{S}_{\text{new}}$  no earlier than it exits the smaller set  $\mathcal{S}_{\text{static}}$ , the proposed strategy strictly

inherits the global MIET lower bound from the static benchmark (i.e.  $\tau_{\text{new}} \geq \tau(\sigma_*) > 0$ ), thereby structurally excluding Zeno behavior.

**Remark 3.2 (Structural Flexibility).** The proposed diagonal structure of  $\bar{\Psi}$  introduces the parameters  $\alpha_x, \alpha_e > 0$  and  $\gamma$  as additional degrees of freedom. Unlike approaches that enforce a rigid spherical triggering geometry, these parameters allow the triggering mechanism to adapt to the ellipsoidal geometry of the Lyapunov function  $V(x) = x^T S x$ . While the satisfaction of the structural constraint depends on  $S$ , this flexibility significantly facilitates the search for feasible triggering policies by exploiting the directional information contained in  $S$ .

### 3.4. Theoretical analysis

In this subsection, we provide rigorous theoretical guarantees for the proposed topology-aware event-triggered control strategy. We establish that the solution to the optimization problem formulated in Sec. 3.3 ensures both the stability of the closed-loop system and the exclusion of the Zeno behavior.

**Theorem 3.3 (Stability and Zeno-freeness).** Consider system (1) with the controller  $u(t) = Kx(t_k)$  designed in Sec. 2. Let  $(\alpha_x^*, \alpha_e^*, \gamma^*)$  be the optimal solution to Problem 3.1, and define the topology-aware triggering rule as

$$t_{k+1} = \inf\{t > t_k : z(t)^T \bar{\Psi}(\alpha_x^*, \alpha_e^*, \gamma^*) z(t) > 0\}. \quad (11)$$

Then, the following properties hold:

- (1) *Global Exponential Stability:* The origin of the closed-loop system is globally exponentially stable.
- (2) *Guaranteed MIET:* The inter-event times  $\{t_{k+1} - t_k\}_{k \in \mathbb{N}_0}$  are globally lower-bounded by the static bound  $\tau(\sigma_*) > 0$  defined in Eq. (7).

**Proof.** *Proof of Item 1 (Stability):* Let  $V(x) = x^T S x$  be the candidate Lyapunov function. From the data-driven representation analysis in Sec. 2.2, the time derivative of  $V(x)$  along the closed-loop trajectories is expressed in terms of the augmented state-error vector  $z(t) = [x(t)^T, e(t)^T]^T$  as

$$\dot{V}(x) = z(t)^T M z(t). \quad (12)$$

In view of LMI (10b), we have  $\mu M - \bar{\Psi} \leq -\epsilon I$  for some scalar  $\epsilon > 0$ , which implies that

$$M \leq \frac{1}{\mu}(\bar{\Psi} - \epsilon I). \quad (13)$$

Substitution of (13) into (12) yields

$$\dot{V}(x) = z(t)^T M z(t) \leq \frac{1}{\mu}(z(t)^T \bar{\Psi} z(t) - \epsilon \|z(t)\|^2).$$

Since the triggering rule (11) ensures that  $z(t)^T \bar{\Psi} z(t) \leq 0$  holds for all  $t \in [t_k, t_{k+1})$ , it follows immediately that

$$\dot{V}(x) \leq -\frac{\epsilon}{\mu} \|z(t)\|^2 \leq -\frac{\epsilon}{\mu} \|x(t)\|^2 \leq -\frac{\epsilon}{\mu \lambda_{\max}(S)} V(x),$$

which implies that the closed-loop system is globally exponentially stable.

**Remark 3.4 (Role of parameter  $\mu$ ).** The scalar  $\mu > 0$  functions as an S-procedure multiplier that incorporates the triggering constraint into the dissipation inequality. As derived in the proof of Theorem 3.3, the LMI condition implies that  $\dot{V}(x) \leq \frac{1}{\mu} z(t)^T \bar{\Psi} z(t)$  (ignoring the strictly negative margin  $\epsilon$ ). This relationship explicitly highlights that  $\mu$  acts as a scaling factor: it weights the triggering threshold  $z(t)^T \bar{\Psi} z(t)$  against the energy decay rate  $\dot{V}(x)$ . In our sequential design, since  $\dot{V}(x)$  is determined by the pre-computed controller, treating  $\mu$  as a free decision variable allows the optimization solver to automatically adjust this scaling to maximize the admissible triggering region (inter-event times) while maintaining stability.

*Proof of Item 2 (Zeno-freeness):* The proof relies on the set-inclusion argument derived from constraint (10c). As established in Sec. 3.3, the condition  $\bar{\Psi} \leq \Psi_{\text{static}}(\sigma_*)$  ensures that the proposed safe set  $\mathcal{S}_{\text{new}}$  contains the safety set  $\mathcal{S}_{\text{static}}$  under the static isotropic triggering. Consider a trajectory starting at  $t_k$  with a reset error  $e(t_k) = 0$ . The time interval until the next trigger is the time required for the trajectory  $z(t)$  to exit the safe set. Since  $\mathcal{S}_{\text{static}} \subseteq \mathcal{S}_{\text{new}}$ , any trajectory must exit  $\mathcal{S}_{\text{static}}$  before (or at the same time as) it exits  $\mathcal{S}_{\text{new}}$ . Therefore, the inter-event time of the proposed method  $T_{\text{new}}$  satisfies  $T_{\text{new}} \geq T_{\text{static}}$ . According to (7), the static interval is lower-bounded by  $\tau(\sigma_*)$ . Thus, we have

$$t_{k+1} - t_k \geq \tau(\sigma_*) > 0,$$

which proves the existence of a global minimum inter-event time and excludes the Zeno behavior.  $\square$

### 3.5. Discussion on performance improvement

While Theorem 3.3 guarantees that the proposed strategy is theoretically “at least as good as” the static isotropic benchmark in terms of Zeno-freeness, the primary motivation for the topology-aware design is to achieve a practical reduction in the communication frequency. This improvement stems from two algebraic mechanisms: geometric expansion via energy shaping and dynamic delay via dissipative coupling.

#### 3.5.1. Geometric expansion mechanism

To explicitly reveal the expansion mechanism, consider the simplified case where  $\gamma = 0$  and  $\alpha_x = \alpha_e = 1$ . In this setting,

our proposed topology-aware strategy enforces the energy-weighted constraint  $e^T S e \leq \sigma_*^2 x^T S x$ .

Using standard eigenvalue bounds, we immediately obtain

$$\lambda_{\min}(S) \|e\|^2 \leq e^T S e \leq \sigma_*^2 x^T S x \leq \sigma_*^2 \lambda_{\max}(S) \|x\|^2.$$

Rearranging these terms yields an effective upper bound on the error magnitude:

$$\|e\|^2 \leq \sigma_*^2 \frac{\lambda_{\max}(S)}{\lambda_{\min}(S)} \|x\|^2 = \sigma_*^2 \kappa(S) \|x\|^2.$$

This result contrasts with the static isotropic condition, which strictly enforces  $\|e\|^2 \leq \sigma_*^2 \|x\|^2$ . The derivation demonstrates that our mechanism amplifies the admissible error bound by a factor up to the condition number  $\kappa(S)$ . This geometric expansion allows the system to tolerate significantly larger errors in directions corresponding to the “slow modes” (associated with small eigenvalues of  $S$ ), where the system is naturally robust.

### 3.5.2. Dynamic discrimination mechanism

The coupling term  $\gamma$  introduces a mechanism to discriminate between “beneficial” and “harmful” errors. The triggering condition implies that

$$\underbrace{-\alpha_x \sigma_*^2 x^T S x + \alpha_e e^T S e}_{\text{Energy terms}} + \underbrace{2\gamma x^T \Phi e}_{\text{Coupling term}} \leq 0.$$

This term acts as a signed modulator of the threshold:

- *Delaying (Beneficial)*: When the error vector  $e$  acts in a stabilizing direction relative to the closed-loop dynamics (specifically,  $x^T \Phi e < 0$ , meaning the error instantaneously contributes to the decay of the Lyapunov function), the negative coupling term cancels out part of the positive error energy penalty. This mechanism effectively “rewards” the beneficial error by pulling the triggering value deeper into the safety region, thereby delaying the transmission to conserve resources.
- *Accelerating (Harmful)*: Conversely, if  $x^T \Phi e > 0$  (i.e. the error opposes stabilization), the term increases the triggering value. This effectively tightens the threshold locally, prompting an earlier update to prevent the accumulation of destabilizing effects.

Unlike decoupled designs that must be conservative to handle the worst-case harmful error, our topology-aware structure dynamically adjusts its tolerance based on the instantaneous impact of the error, achieving a superior trade-off between efficiency and robustness.

### 3.6. Synthesis algorithm

The complete data-driven design procedure is summarized in Algorithm 3.5. This algorithm outlines the offline

computational steps required to synthesize the controller gain and the topology-aware triggering matrix directly from the collected dataset.

#### Algorithm 3.5.

**Require:** Offline dataset  $\{U_0, X_0, X_1\}$ .

**Ensure:** Controller gain  $K$  and triggering matrix  $\bar{\Psi}^*$ .

- 1: **Step 1: Initial Stabilization and Learning**
- 2: Solve the LMI in Sec. 2.3 to obtain the variable  $Y$ .
- 3: Compute controller  $K \leftarrow U_0 Y (X_0 Y)^{-1}$  and Lyapunov matrix  $S \leftarrow (X_0 Y)^{-1}$ .
- 4: Solve the static isotropic design ([26, Theorem 2]) to find the optimal isotropic parameter  $\sigma_*$ .
- 5: **Step 2: Structural Matrix Construction**
- 6: Compute the data-based auxiliary matrix  $L \leftarrow [U_0^T, X_0^T]^\dagger [K^T, 0]^T$ .
- 7: Construct the coupling matrix  $\Phi \leftarrow S X_1 L$  using the definition in Eq. (8).
- 8: Construct the dissipation matrix  $M$  using Eq. (6).
- 9: **Step 3: Topology-aware Optimization**
- 10: Formulate **Problem 3.1** with the objective: maximize  $\alpha_x$ .
- 11: Solve the SDP to obtain the optimal parameters  $\alpha_x^*, \alpha_e^*, \gamma^*$ .
- 12: **Step 4: Trigger Construction**
- 13: Assemble the final generalized triggering matrix:

$$\bar{\Psi}^* \leftarrow \alpha_x^* \begin{bmatrix} -\sigma_*^2 S & 0 \\ 0 & 0 \end{bmatrix} + \alpha_e^* \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix} + \gamma^* \begin{bmatrix} 0 & \Phi \\ \Phi^T & 0 \end{bmatrix}.$$

- 14: **Result:** The computed controller  $K$  and the optimized triggering matrix  $\bar{\Psi}^*$ .

**Remark 3.6 (Optimal Benchmark  $\sigma_*$ ).** Note that  $\sigma_*$  is obtained by solving the optimization problem in [26, Theorem 2] to maximize the triggering threshold, serving as the optimal static benchmark for comparison. Specifically,  $\sigma_*$  represents the theoretical upper bound on the relative error achievable by isotropic triggering rules under the required stability constraint. In this work, we utilize  $\sigma_*$  as a baseline to evaluate the proposed topology-aware mechanism, demonstrating that the proposed method can further reduce communication load even when compared against the optimal static setting.

**Remark 3.7 (Computational Complexity).** It is important to note that Algorithm 3.5 represents an offline synthesis procedure. While solving the SDP entails computational complexity depending on the system dimension, this optimization is performed only once prior to deployment. In contrast, the online execution only involves evaluating the triggering condition (Eq. (11)), which consists of standard matrix-vector multiplications. Consequently, the computational burden of the SDP does not affect real-time

performance, facilitating the implementation of the proposed framework in practical control applications.

#### 4. Numerical Experiments

In this section, we validate the effectiveness of the proposed generalized topology-aware strategy through a numerical example. We compare our method against the static isotropic benchmark (e.g. [26]) to demonstrate the performance gains achieved by the decoupled structural optimization.

##### 4.1. System setup and performance validation

We consider an unstable continuous-time linear system defined by the following system matrices:

$$A = \begin{bmatrix} 5 & 6 \\ -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The open-loop system is unstable with eigenvalues at  $\{4, -1\}$ .

The data collection phase is conducted on the open-loop system. To ensure reproducibility, the random input sequence is generated with a fixed seed (MATLAB setting `rng(1)`). The input is uniformly distributed, and state measurements are collected with a sampling period of  $T_s = 0.1$  s. A total of  $T = 10$  samples are recorded to construct the data matrices  $\{U_0, X_0, X_1\}$ . Despite the small sample size, the richness assumption (Assumption 2.1) is verified to hold, ensuring the data are sufficient to represent the system dynamics.

Based on the collected data, we first synthesize the stabilizing controller using the standard data-driven LMI, yielding the feedback gain  $K = [-7.0000, -5.0000]$ . Following the procedure in [26], the optimal static triggering parameter is calculated as  $\sigma_* = 0.2325$ , which establishes a theoretical MIET of  $\tau(\sigma_*) = 0.0219$  s.

Subsequently, we solve Optimization Problem 3.1 to obtain the proposed topology-aware parameters. The solution yields

$$\alpha_x^* = 1.6209, \quad \alpha_e^* = 0.0014, \quad \gamma^* = 0.0219.$$

Equipped with these synthesized parameters, we simulate the closed-loop system to evaluate the runtime performance.

The state trajectories  $(x_1, x_2)$  for both methods are depicted in Fig. 2. Despite the relaxed triggering conditions implied by the optimized parameters, the proposed topology-aware controller (dashed lines) maintains smooth asymptotic convergence to the origin, virtually indistinguishable from the static isotropic strategy (solid lines). This confirms that the reduction in communication does not come at the cost of weaker closed-loop transient performance.

The efficiency of the proposed method is explicitly highlighted in Fig. 3 and summarized in Table 1. Over the 10 s simulation window, the static isotropic method triggers 121

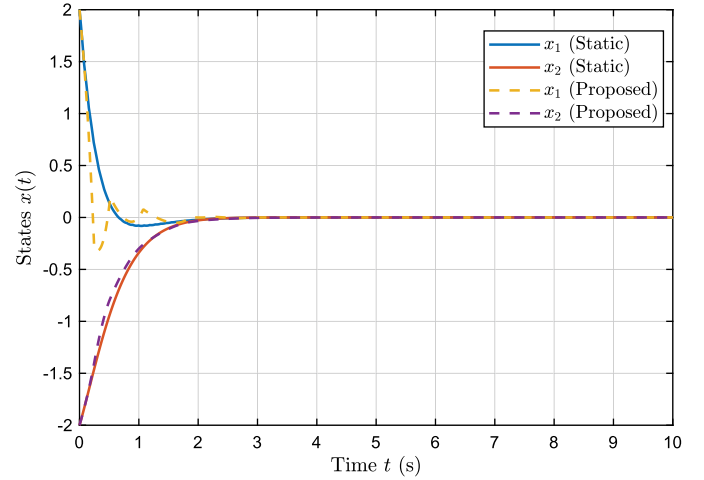


Fig. 2. State trajectories of the closed-loop system under static and topology-aware triggering.

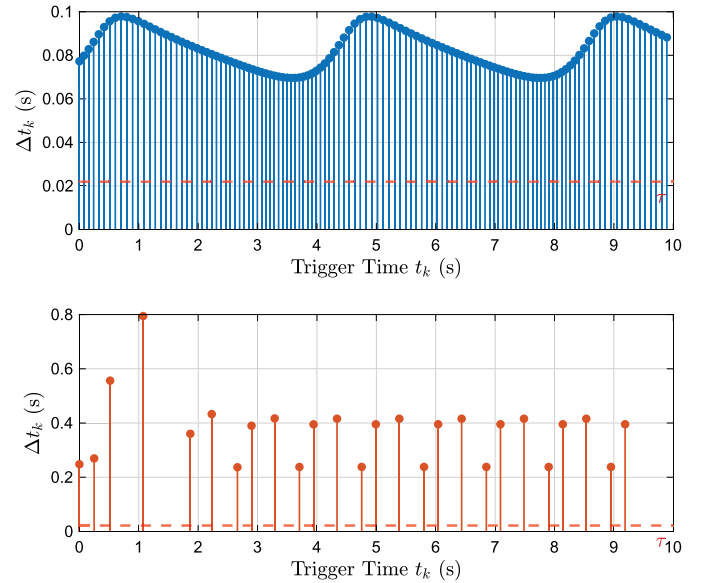


Fig. 3. Comparison of inter-event times: static isotropic strategy (top) and proposed topology-aware strategy (bottom).

Table 1. Performance comparison ( $T_{\text{sim}} = 10$  s).

| Method                | Events | Min. $\Delta t$ (s) | Avg. $\Delta t$ (s) |
|-----------------------|--------|---------------------|---------------------|
| Static isotropic [26] | 121    | 0.0696              | 0.0825              |
| Proposed              | 26     | 0.2369              | 0.3686              |

Note: Theoretical lower bound  $\tau(\sigma_*) = 0.0219$  s.

times. In contrast, the topology-aware method triggers only 26 times, achieving a substantial 78.5% reduction in network traffic.

Furthermore, analyzing the inter-event times  $\Delta t_k = t_{k+1} - t_k$  in Fig. 3 reveals a significant improvement in sampling behavior. While the theoretical lower bound is

$\tau = 0.0219$  s (indicated by the red dashed line), the minimum interval observed for the proposed method is  $0.2369$  s — more than triple the minimum interval of the benchmark ( $0.0696$  s). This demonstrates that the proposed structural decomposition effectively eliminates unnecessarily frequent updates even in worst-case scenarios.

## 5. Conclusions

In this paper, we have presented a novel topology-aware framework for data-driven event-triggered control of unknown linear systems. Central to our approach is the dual utilization of offline data, which is leveraged to synthesize not only the feedback controller but also a structure-embedded triggering mechanism. By decomposing the triggering condition into state threshold scaling, error sensitivity weighting, and dissipative coupling, we reshaped the safety region to align with the system's natural energy landscape. This approach achieves significant communication savings while strictly guaranteeing global exponential stability and Zeno-freeness. The proposed framework provides a foundation for broader control scenarios. Future work will focus on extending this topology-aware paradigm to output-feedback settings and nonlinear systems, as well as incorporating robustness against noise. Additionally, developing online learning mechanisms to adapt the controller and triggering logic for time-varying dynamics represents a promising direction for further research.

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