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Design and Modeling of a Quadcopter with Double Axis Tilting Rotors

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Multirotors are well suited for application tasks such as surveillance and exploration of otherwise inaccessible areas. Standard quadrotors have limitations in their possible configurations due to their underactuation. For this reason, some spatial configurations are not possible, such as hovering while maintaining a nonhorizontal orientation. This paper presents an overactuated quadrotor platform with double axes tilting propellers. The peculiarity of the proposed platform is that, beside the usual control on the four propellers, it allows to tilt each arm where motors are mounted along two independent axis. The resulting number of control inputs is 12, allowing a higher number of stable configurations with respect to traditional quadrotors. As a result, it can assume spatial orientations that are not possible for traditional quadcopters, enabling the possibility to deal with obstacles that would generally impede the motion of normal quadcopters. This feature allows to potentially explore a larger space. This paper presents the design and modeling of the quadrotor. Numerical simulations are carried out to show the effectiveness of the proposed solution.

Keywords: UAV; quadrotors; design; modeling; mobile robotics.

1. Introduction

Quadrotors are a class of flying vehicles capable of vertical take off and landing (VTOL) thanks to their actuation based on four rotors that provide high stability and manoeuvrability. This feature, together with the advances in embedded systems and sensing technology in recent years, has stimulated the research on quadrotors in different application domains, including entertainment [20], agriculture [17] and construction [4]. The result is a great variety of shapes and dimensions, from small low-cost toy quadcopters to more unusual solutions, like tricopters to octacopters, i.e. multirotors equipped with 3 and 8 rotors, respectively. The report in [23] estimates that the market for civil Unmanned Aerial Vehicles (UAV) is expected to constantly grow in the upcoming decade, reaching in 2025 four times the present market.

In recent years, autonomous robots for Search And Rescue (SAR) applications raised an increasing interest [11–13, 26]. In this domain, multicopters play a major role [3, 15, 18]. In challenging situations, the quadrotor is required to fly in a working space populated by obstacles with irregular shapes and reduced manoeuvrability space. For instance, passages or openings in a wall may be large enough for flying through only if the quadrotor is suitably tilted along the roll axis. To allow a traditional quadrotor to deal with these situations, controllers capable of aggressive manoeuvres have been investigated [5, 14, 27]. Although this approach represents an appealing research challenge from the control viewpoint, its main practical drawback is that a quadrotor requires additional time and space to return to a stable configuration after the manoeuvre operated in tilted orientation. In worst conditions, in fact, this approach cannot guarantee the necessary obstacle avoidance capability due to little manoeuvrability space.

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A viable approach to this problem is the development of a quadrotor that is able to maintain a tilted orientation in a stable manner. However, for traditional quadrotors, some state space configurations are impossible to be maintained firmly due to their intrinsic underactuation. To address this issue, some possible approaches are proposed in the literature. One possible solution is to increase the number of propellers, while setting their orientation such that their propelling forces are not constrained to the vertical axis only. This approach is used in some hexrotors [8, 9, 22] and in some configurations resulting from the solution of optimization problems [2, 19]. Positive aspects of this approach include, beside the addressed underactuation, a rigid mechanical structure and the minimization of the number of actuators, making the solution simple and less prone to breakage. On the other hand, a relevant drawback of this method is the inherent low hovering efficiency, which is not constant in all the configurations, i.e., to obtain arbitrarily directed total forces and torques, part of the propeller thrust is canceled due to force components oriented in opposite directions. Therefore, part of the energy is wasted to generate the overall resulting force in the desired direction.

The alternative option considered in this paper solves the underactuation issue by dynamically changing the orientation of the propellers. With this solution, the propellers can be oriented towards the direction of the actual desired total force. The researches in [1, 6, 21, 24, 25] provide exhaustive analyses of quadcopters with single axis tilting propellers. When the vehicle flies with the propeller's plane parallel to the ground, this architecture allows to control the propeller orientations to avoid mutual contrast. Therefore, the propeller thrusts are fully exploited with no efficiency loss. However, in every configuration different from the horizontal hovering, the propellers are operated such that their orientation is different with respect to the actual direction of total thrust. Therefore, they partially work against each other, decreasing the efficiency of the solution.

To avoid this issue, this paper proposes a quadrotor with two tilting axes on each arm. Hence, every motor can be independently oriented towards the desired direction of thrust and assume an arbitrary orientation while both hover and move. This leads to some advantages with respect to traditional multicopters, e.g. the ability to fly in working spaces that are not accessible to standard quadrotors, such as with passages narrower than the vehicle minimum width, where the vehicle has to assume a disadvantageous attitude. The contribution of this paper is the modeling of the system and the analysis of geometrical properties that allow the vehicle to fly across narrow passages. This analysis is supported by the simulation of the physical features of the vehicle while performing a manoeuvre through a narrow tunnel. The control of the vehicle is done by means of simple PID controller, with feedback on positions and angles, and a wrench generation technique that constrains the rotors to be oriented along the desired direction. Although more sophisticated control techniques may lead to better results in terms of configuration control and stabilization, the main focus of this paper is on design and modeling of the overall system and not on the design of the controller itself.

This paper is organized as follows. The dynamical model of quadrotor with tilting arms and motors is presented in Sec. 2. In the same section, the elaborated model is used to derive the dynamics of a quadrotor with a single tilting axis and a standard quadrotor. In Sec. 3, the geometrical analysis and the advantages with respect to traditional multirotors in passing through narrow tunnels (Sec. 3.1) and windows (Sec. 3.2) are described. Section 4 presents the developed simulation framework, while in Sec. 5, simulation results are carried out to assess the feasibility of the quadrotor structure. Finally, Sec. 6 states our conclusions.

2. System Model

The peculiarity of the system presented in this paper differs from standard quadcopters by the fact that, in addition to the propeller rotation velocity, the propeller orientation can be arbitrarily controlled. More specifically, it is possible to independently rotate each propeller along two axes, as shown in Fig. 1. This solution allows to point each propeller's thrust towards the desired direction. Therefore, there is the possibility to maintain stable flying configurations that standard quadcopters can only achieve in dynamic behaviors, e.g. when the orientation of the propellers is not parallel to the gravity vector.

Figure 2 shows the practical realization of the joints. They are implemented using servomotors directly coupled

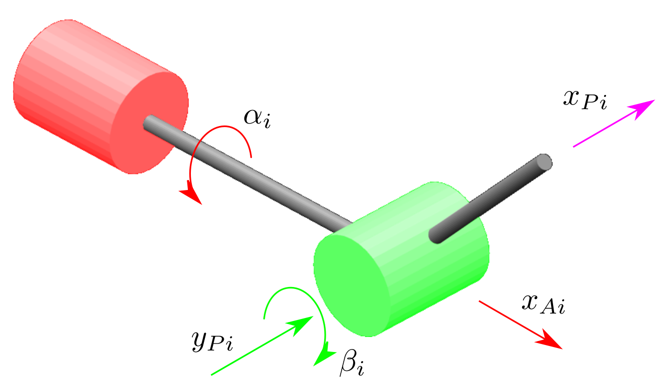

Fig. 1. Mechanical scheme of the joints that allow arbitrary orientation of the propellers. Angles α_i and β_i are the orientations around the two axis, x_{Ai} and y_{Pi}, respectively.

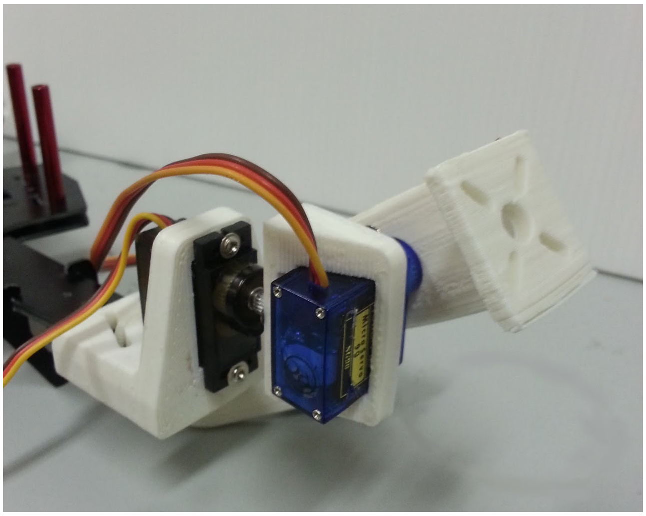

Fig. 2. Realization of the joints by means of servomotors and 3D printed mechanical parts.

to 3D printed links. The propeller motor is mounted at the end of the rightmost part.

The remainder of this section presents and analyzes the dynamics of the proposed system.

2.1. *Definitions and notations*

The different frames that are necessary to describe the system include the inertial frame, the body local frame and the ones related to the arm and propeller rotations. Let $F_W : \{O_W, x_W, y_W, z_W\}$ be the world inertial frame and $F_B : \{r_B, x_B, y_B, z_B\}$ be the moving frame attached to the body of the quadcopter. The transformation from F_W to F_B is given by a vector of motion r_B and a rotational matrix R_B^W. Linear and angular velocities of the quadrotor body represented in the body frame F_B are denoted as $\mathbf{v}_B$ and ω_B, respectively.

Let $F_{Ai} : \{r_{Ai}, x_{Ai}, y_{Ai}, z_{Ai}\}$ be the frame of the ith arm, rotated from F_B of an angle α_i around the x_{Ai} axis, such that z_B is parallel to z_{Ai} when $\alpha_i = 90°$. Figure 3 shows a graphical representation of the aforementioned frames.

In general, the relationship between the body frame F_B and the arm frames F_{Ai} could be divided into two main transforms: a fixed one between the body frame F_B and the arm joint, and a second one between the arm joint and the arm, which depends from β_i. By assuming to consider only quadrotors with axial-symmetric configuration, the first transform can be omitted and the resulting rotational matrix R_{Ai}^B is given by

$$R_{Ai}^B = R_Z\left((i-1)\frac{2\pi}{N}\right)R_X\left(\alpha_i - \frac{\pi}{2}\right), \tag{1}$$

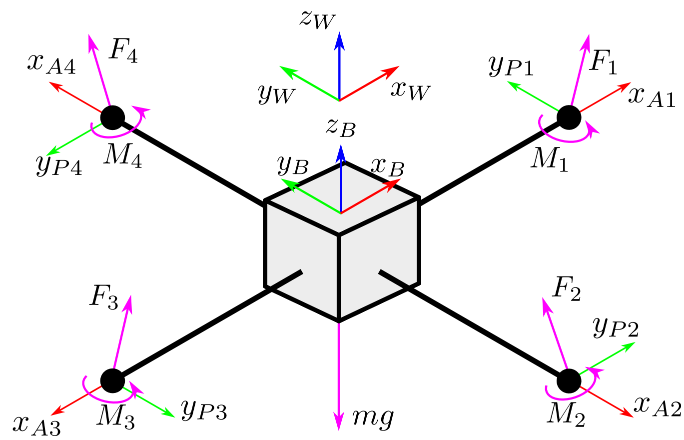

Fig. 3. Diagram showing the main frames and forces acting on the quadcopter. The center of mass is assumed to be in the body frame center. F_i and M_i are the forces and torques associated to the ith propeller.

where N is the number of arms. The terms $R_X(\zeta)$, $R_Y(\zeta)$ and $R_Z(\zeta)$ denote the canonical rotations along the X-, Y- and Z-axes of angle ζ.

Since a specific case of quadrotor (i.e. $N = 4$) is considered in this paper, Eq. (1) becomes

$$R_{Ai}^B = R_Z\left((i-1)\frac{\pi}{2}\right)R_X\left(\alpha_i - \frac{\pi}{2}\right). \tag{2}$$

Similarly, the position of the second tilting joint is at r_{Ai}:

$$r_{Ai}^B = R_{Ai}^B\mathbf{L}, \tag{3}$$

where $\mathbf{L}$ is the distance between the arm joint and the motor joint.

Finally, let $F_{Pi} : \{r_{Pi}, x_{Pi}, y_{Pi}, z_{Pi}\}$ be the frame associated to the ith propeller, rotated from F_{Ai} of an angle β_i around the y_{Pi} axis, such that x_{Pi} is parallel to z_{Ai} for $\beta_i = 90°$. The resulting matrices of orientation (r_{Pi}^{Ai}) and position (R_{Pi}^{Ai}) of the propellers are

$$R_{Pi}^{Ai} = R_Y(\beta_i - \pi), \tag{4}$$

$$r_{Pi}^{Ai} = R_{Pi}^{Ai}\mathbf{l}, \tag{5}$$

where $\mathbf{l}$ is the distance between the motor joint and the point where the motor is attached.

The offset $\frac{\pi}{2}$ and π on α_i and β_i, respectively, in Eqs. (1) and (4), have been set to harmonize the modeling of all orientations: when $\alpha_i = \beta_i = 90°$, the equations describe a traditional (i.e. nontilting) quadrotor.

To simplify the presentation, the vectors $\mathbf{L}$ and $\mathbf{l}$ have been defined by modeling the x displacement only, i.e.

$$\mathbf{L} = \begin{bmatrix} L_x \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{l} = \begin{bmatrix} l_x \\ 0 \\ 0 \end{bmatrix}. \tag{6}$$

If vectors **L** and **l** have nonnull components along x_{Ai} and x_{Pi} direction, respectively, the propeller thrusts will induce torques around these axes even in static equilibrium conditions. For the sake of energy saving, these components should be avoided. In this work, any displacement with nonnull components in any direction different from x is considered as a modelling error, which needs to be compensated by the controller.

2.2. *Dynamic equations*

This section derives the equations of the quadcopter motion considering forces and torques applied to the vehicle by the propellers. Any aerodynamic effect different from the ones affecting the propellers, and thus determining the thrust and torques, is neglected. They are considered as errors by the controller. Similarly, the torques generated by the rotation of arms and motors (the second derivative of α_i and β_i, respectively) are neglected and considered as disturbances. This is acceptable in case the inertia of the arms is sufficiently small and the dynamics sufficiently high with respect to the quadrotor ones.

Let $F_{B'i} : \{r_B, x_{B'}, y_{B'}, z_{B'}\}$ the reference frame of the ith axis centered in r_B and $x_{B'}$ parallel to the x_{Ai} of the ith axis. Hence, the rotation $R^B_{B'i}$ is the same $R_Z((i-1)\frac{\pi}{2})$ term used in Eq. (2).

The transformations between the body frame F_B and the ith propeller can be expressed as follows[a]:

$$R^B_{Pi} = R^B_{B'i}\begin{pmatrix} -c\beta_i & 0 & -s\beta_i \\ c\alpha_i\, s\beta_i & s\alpha_i & -c\alpha_i\, c\beta_i \\ s\alpha_i\, s\beta_i & -c\alpha_i & -s\alpha_i\, c\beta_i \end{pmatrix}, \tag{7}$$

$$r^B_{Pi} = R^B_{B'i}\begin{pmatrix} L - c\beta_i\, l \\ c\alpha_i\, s\beta_i\, l \\ s\alpha_i\, s\beta_i\, l \end{pmatrix}. \tag{8}$$

The equations of motion are described in terms of translation and rotational velocities represented in the body frame F_B, $\mathbf{v}_B$ and $\boldsymbol{\omega}_B$:

$$m\dot{\mathbf{v}}_B = F_{\text{ext}} - \boldsymbol{\omega}_B \times (m\mathbf{v}_B), \tag{9}$$

$$J\dot{\boldsymbol{\omega}}_B = M_{\text{ext}} - \boldsymbol{\omega}_B \times (J\boldsymbol{\omega}_B), \tag{10}$$

where F_{ext} and M_{ext} indicate the external forces and torques, while m and J are, respectively, the mass and the inertia tensor.

The F_{ext} vector is composed by the gravity force and the propeller forces represented in body frame F_B:

$$F_{\text{ext}} = R^W_B m\mathbf{g} + \sum_{i=1}^{4} F_i, \tag{11}$$

where $\mathbf{g}$ is the gravity vector in the world frame F_W, and F_i are the propeller forces referred to the body frame.

Assuming that the ith propeller force in the propeller frame F_{Pi} is composed only by a component in the x_{Pi} direction and calling its magnitude F_{Ti}, the term F_i can be expressed as

$$F_i = R^B_{B'i}\begin{pmatrix} -c\beta_i \\ c\alpha_i\, s\beta_i \\ s\alpha_i\, s\beta_i \end{pmatrix} F_{Ti} \tag{12}$$

and

$$F_{Ti} = k\,\omega_i|\omega_i|, \tag{13}$$

where k is the lift constant and ω_i is the angular velocity of the ith propeller.

It is interesting to note that the quadcopter is capable of pointing the force of the ith propeller in the x_{Ai} and y_{Ai} directions by means of the angles β_i and α_i, respectively.

The M_{ext} vector can be decomposed in the torques caused by the propeller forces (M_{Fi}) and by the propeller torques themselves (M_{Mi}), as in

$$M_{\text{ext}} = \sum_{i=1}^{4} M_{Fi} + \sum_{i=1}^{4} M_{Mi}. \tag{14}$$

Similarly, assuming that the ith propeller torques in the propeller frame F_{Pi} has only one component along the x_{Pi} direction, and denoting with M_{Ti} such a torque, the terms M_{Fi} and M_{Mi} can be expressed as follows:

$$M_{Fi} = R^B_{B'i}\begin{pmatrix} 0 \\ s\alpha_i\, s\beta_i((c\beta_i l_x - L_x) - c\beta_i l_x) \\ c\alpha_i\, s\beta_i((L_x - c\beta_i l_x) + c\beta_i l_x) \end{pmatrix} F_{Ti}, \tag{15}$$

$$M_{Mi} = R^B_{B'i}\begin{pmatrix} -c\beta_i \\ c\alpha_i\, s\beta_i \\ s\alpha_i\, s\beta_i \end{pmatrix} M_{Ti} \tag{16}$$

and

$$M_{Ti} = b\omega_i|\omega_i| + I_P\dot{\omega}_i, \tag{17}$$

where b is the drag coefficient and I_P is the scalar inertia of the rotor and the propeller, respectively.

The above equations imply that the ith propeller can cause a torque along the x_{Ai} direction only by means of M_{Ti}.

2.3. *Comparison of different architectures in hover manoeuvres*

It is worth while to note that the presented equations of motion are applicable to multicopters having an arbitrary number of arms and a maximum number of two joints. Indeed, it is possible to describe such multicopters by

[a] In the remainder of this paper, the terms $\sin\zeta$, $\cos\zeta$ and $\tan\zeta$ will be represented as $s\zeta$, $c\zeta$ and $t\zeta$ for brevity.

changing the value of N in Eq. (1) or to set α_i and β_i to constant values in case there is no possibility to tilt the arm or the motor, respectively. This section derives the equation of motion of a single tilting axis copter and a traditional quadcopter starting from the equations presented in the previous sections and compares the architectures.

2.3.1. *Single tilting axis and traditional quadcopter dynamics*

A single axis tilting rotor multicopter can be modeled using the equations of motion presented in Secs. 2.1 and 2.2 by setting $\beta_i = 90°$ or $\alpha_i = 90°$. We will only consider the first case for brevity. In practice, this means that the propellers are perpendicular to the arm along the x_{Ai} axis, leading to

$$F_i = R^B_{B'i} \begin{pmatrix} 0 \\ c\alpha_i \\ s\alpha_i \end{pmatrix} F_{Ti}, \tag{18}$$

$$M_{Fi} = R^B_{B'i} \begin{pmatrix} 0 \\ -L\, s\alpha_i \\ L\, c\alpha_i \end{pmatrix} F_{Ti}, \tag{19}$$

$$M_{Mi} = R^B_{B'i} \begin{pmatrix} 0 \\ c\alpha_i \\ s\alpha_i \end{pmatrix} M_{Ti}, \tag{20}$$

where the forces F_i are now null in the x_{Ai} direction. Hence, to generate a force in that direction, now the vehicle has to rotate the arms on the perpendicular axis of an α_i angle, or it has to pitch itself around it.

A traditional multicopter, instead, can be modeled using Eq. (2) by setting also $\alpha_i = 90°$. Thus, the equations becomes

$$F_i = R^B_{B'i} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} F_{Ti}, \tag{21}$$

$$M_{Fi} = R^B_{B'i} \begin{pmatrix} 0 \\ -L \\ 0 \end{pmatrix} F_{Ti}, \tag{22}$$

$$M_{Mi} = R^B_{B'i} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} M_{Ti}, \tag{23}$$

where the only way to move in the x_W and y_W directions is to change the attitude. This means that there is no possibility to hover in nonhorizontal attitude.

2.4. *Comparison in hover manoeuvre*

As aforementioned and depicted in Fig. 4, a traditional quadcopter has no means of hovering in a nonhorizontal attitude, since the propeller thrust are constrained to be perpendicular to the body itself. In case of stable horizontal hovering, however, the thrust on the single motor is exactly $F_{Ti} = mg/4$, where g is the gravity and m the mass of the vehicle. Since all the thrust is directed towards the gravity force, this is the most efficient configuration.

As depicted in Fig. 4, the dynamic equations of a single tilting axis quadcopter hovering in stable nonhorizontal attitude can be expressed as

$$2F_{T1} + 2F_{T2} = m\bar{g} \tag{24}$$

and decomposed along the z_W and y_W direction as

$$\begin{aligned} 2|F_{T1}|\sin(\phi + \beta) + 2|F_{T2}|\sin(90 + \phi) &= mg, \\ 2|F_{T1}|\cos(\phi + \beta) + 2|F_{T2}|\cos(90 + \phi) &= 0, \end{aligned} \tag{25}$$

where ϕ is the roll angle and β is the angle of rotation of the propellers. α instead is set to 90°.

From the second equation, it is possible to see how, in order to compensate for the body rotation, part of the propellers thrust will be in conflict, thus increasing the energy consumption.

Using F_H and F_T, defined as

$$\begin{aligned} F_H &= 2|F_{T1}| \cos\phi + \beta, \\ F_T &= 2|F_{T1}| + 2|F_{T2}| \end{aligned} \tag{26}$$

as means of comparisons, Fig. 5 shows the profile of the two values as ϕ and β change for a quadcopter of 1 kg in the case of single tilting or double tilting axes. The analysis has been made solving Eq. (25) and the figure shows the values of F_H and F_T with ϕ isolines, i.e. different attitude, and different β. The red line, instead, is the value obtained for the two axis quadcopters.

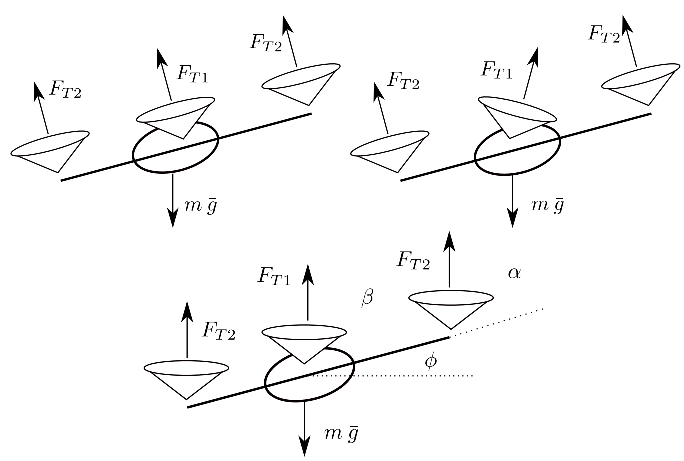

Fig. 4. Comparison between traditional, single and double tilting axis quadrotor free body diagrams in hovering. The horizontal components of the involved forces are not null for all the configurations but the double axis one. In the third figure, the angles used in the comparison are shown.

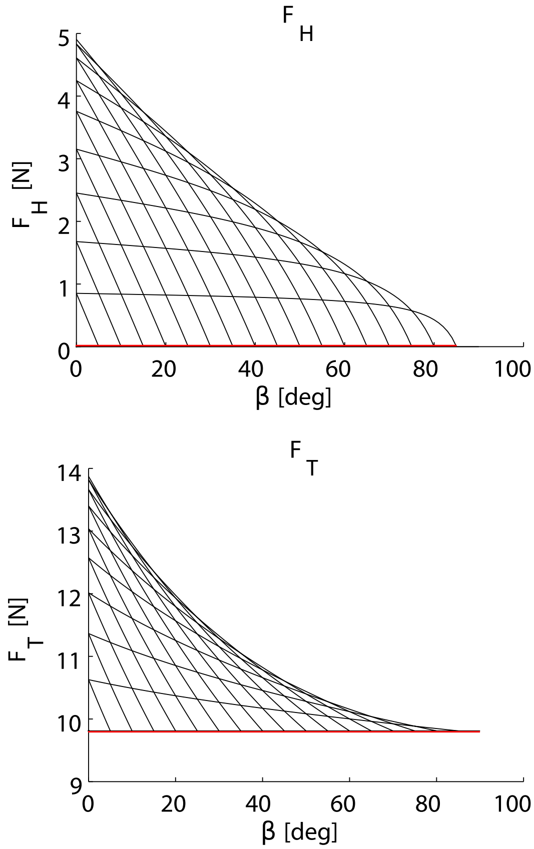

Fig. 5. Comparison between single (black lines) and double (red line) tilting axis quadrotors in hovering: F_H is the horizontal components of the involved forces and F_T is the total thrust, as defined in Eq. (26).

It is worth while to note that, for every configuration of ϕ and β, the horizontal component of the thrusts is not null, but in the case of $\phi + \beta = 90^\circ$. This leads to a value of the total thrust F_T higher than the weight of the system. Thus, there is an intrinsic energy loss, causing a loss in the autonomy itself of the vehicle.

Moreover, the more the thrust needed, the more the current drained: since the electrical system is not linear, an higher current means an even more fast discharge of the battery pack. This means that a strategy where $\beta = 90 - \phi$, and consequently $|F_{T2}| = 0$, could not be optimal. Indeed, in this case $|F_{T1}| = mg/2$ anyway, needing the motors to be sized for higher power, meaning higher weights and costs.

The solution this paper proposes, instead, can orientate the propellers always along the z_W direction, even when the attitude is not horizontal. This means that it can hover as a traditional quadcopter, maximizing the efficiency of the thrusts.

In the literature, tilting rotors have also been used with different architectures than quadcopters, like bicopters [7,10], i.e. vehicles with two propellers that rotate in opposite directions. In this case, the pitch angle can be controlled exploiting both the moment of inertia of the propellers and the drag moment on the propeller. While the first one can be used only in dynamic manoeuvres, the second one is caused by the propeller rotational speed itself and depends only by the angle α_i of the propeller. This means that, in order to have a pitch moment, the propellers need to be rotated, causing the thrusts to conflict, as seen in the previous analysis.

3. Geometrical Analysis of Quadcopter Operations

This section analyzes the quadcopter with double axes tilting propellers while flying across a narrow passage. In particular, the geometrical considerations related to this specific case are analyzed.

3.1. *Flying across a rectangular tunnel*

While it is possible for traditional quadcopters to fly across a large window in any desired attitude configuration, the task of flying across a relatively long and narrow tunnel with arbitrary orientation is challenging. In fact, traditional quadcopters can only hover in horizontal orientation; otherwise, longitudinal velocities and accelerations become non-null, and additional space would be required to travel in tilted orientation. This fact limits the possibility of a traditional quadrotor to fly along tunnels where manoeuvres need in place tilting operations or across openings requiring stable fly in tilted orientation.

Let us consider a rectangular duct of height h and width w, and denote the orientation of the duct with ϕ_g, θ_g and ψ_g. The most convenient and safe way to approach the duct is to head to its opening with a velocity directed towards the longitudinal axis of the tunnel and a yaw angle of 45°. As we can see from Fig. 6, in this condition, the width occupation of the quadrotor is $\sqrt{2}\,L_x + d$ instead of $2\,L_x + d$, where d is

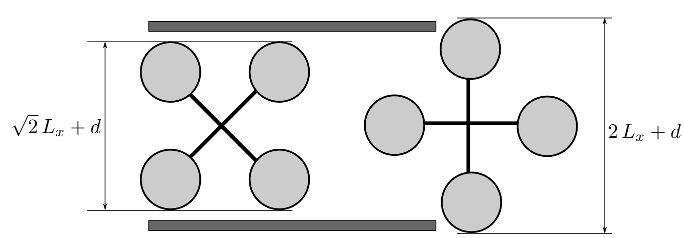

Fig. 6. Diagram of the quadrotor flying through the tunnel. As can be seen, the orientation that minimizes the width of the quadcopter is the one on the left, where the quadcopter is rotated of 45° around the z_B axis.

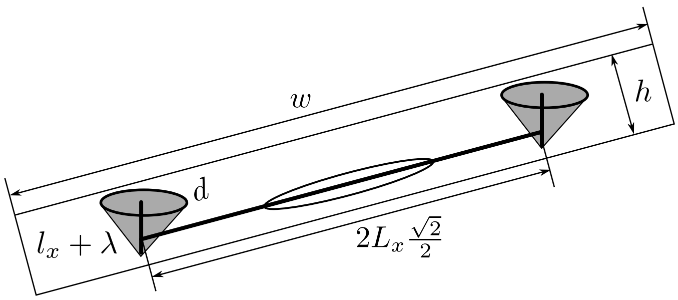

Fig. 7. Diagram of a quadcopter with double axes tilting propeller flying through a narrow space. It is assumed that the body of the quadcopter is parallel to the opening.

the propeller diameter, and all the motors can be used for the forward movement. As a result, the problem translates to flying across a horizontal rectangular tunnel, as shown in Fig. 7, facing a gravity force directed along $R_W^B\ z_B$. Note that a traditional quadcopter cannot face a gravity vector whose direction is not along z_B since it would drift instead of hovering. In that case, the tunnel has to be large enough to permit a standard quadrotor to fly through it with acceptable speed. Figure 8 shows the difference between a standard and a tilting propeller quadcopter while flying through a nonhorizontal tunnel in equilibrium conditions, i.e. when the propellers are directed towards the gravity vector.

To carry out an accurate evaluation of the maneuvering capabilities of the proposed solution, each rotor is modeled as a cone. The base of each cone corresponds to the propeller; it has diameter d and its edge is located at the end of the propeller arm, i.e. its height is $l_x + \lambda$, where λ is the distance from the propeller to the lowest point in direction z_{Pi}, e.g. a landing gear.

It is assumed that d, l_x and λ are greater than the body hull height. Therefore, for the sake of obstacle avoidance, the whole vehicle can be modeled as four cones located at r_{Pi}. Figures 9 and 10 show a graphical representation of aforementioned geometrical quantities.

To ensure that the quadcopter can fly across the opening, the two cones, rotated by α_i and β_i according to the desired velocity, are required to pass through the opening itself.

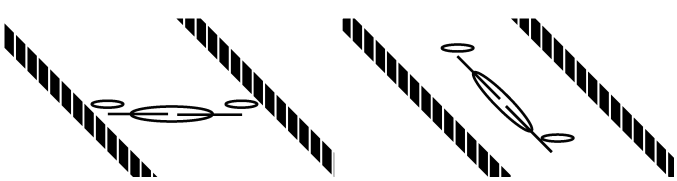

Fig. 8. Comparison between a standard quadcopter (left image) and a tilting propeller quadcopter (right image) passing through a nonhorizontal tunnel.

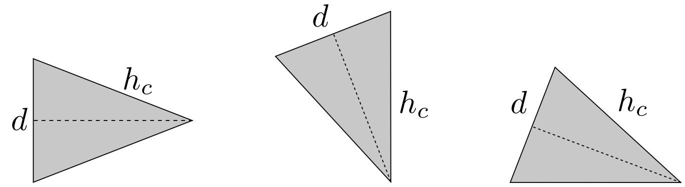

Fig. 9. Different configurations of the cone of the propeller depending on the orientation in case of $\|h_c\|$ greater than d. The straight dashed line is the Z_{Pi}-axis.

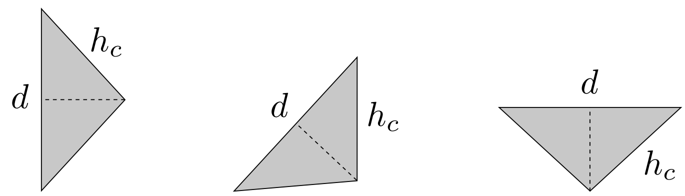

Fig. 10. Different configurations of the cone of the propeller depending on the orientation in case of d greater than $\|h_c\|$. The straight dashed line is the Z_{Pi}-axis.

Denoting with h_c the vector described by the intersection of the plane formed by x_{Pi} and the direction of motion and the cone, in the reference frame defined by F_{Pi} the value of h_c can be calculated as

$$h_c = \begin{pmatrix} l_x + \lambda \\ 0 \\ -\dfrac{d}{2} \end{pmatrix}. \tag{27}$$

The height $\bar{h}$ of the opening must be greater than $\max(\|h_c\|, d)$ to let the cone pass through while it is in its worst configuration. In this case, the propellers have no constraints on their motion and the quadcopter can follow any velocity profile maintaining its attitude.

On the other hand, if the $h < \bar{h}$, the motion is allowed only for some possible values of α_i and β_i. In the remainder of this section, only the case $\|h_c\| > d$ will be considered, since the case where $\|h_c\| \leq d$ can be analyzed with the same approach. Hence, h should be greater than the cone height $\hat{h}_i$ at hovering configuration $\tilde{\alpha}_i$ and $\tilde{\beta}_i$. Therefore, the following

$$\hat{h}_i = R_X\left(\tilde{\alpha}_i - \frac{\pi}{2}\right) R_Y(\tilde{\beta}_i - \pi) R_X(\hat{\theta}) h_c \cdot \hat{z}_B, \tag{28}$$

where $\hat{z}_B$ is the unit vector along the z_B axis, and $R_X(\hat{\theta})$ is the angle rotation that maximizes $\hat{h}_i$. The values $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ can be calculated as

$$\begin{pmatrix} \tilde{\alpha}_i \\ \tilde{\beta}_i \end{pmatrix} = \begin{pmatrix} \dfrac{\pi}{2} \\ \pi \end{pmatrix} + R_Z(\psi) R_{B'i}^B \begin{pmatrix} \phi \\ \theta \end{pmatrix}, \tag{29}$$

where ϕ, θ and ψ are the roll, pitch and yaw angle, respectively, such that the thrust faces the gravity vector. The value of $\hat{\theta}$ can be calculated as follows:

$$\hat{\theta} = -\arctan\left(\frac{1}{t\tilde{\alpha}_i\, c\tilde{\beta}_i}\right), \tag{30}$$

which allows to derive the value of $\hat{h}_i$ given $\tilde{\alpha}_i$ and $\tilde{\beta}_i$, if $\tilde{\beta}_i \neq 0$. If $h < \hat{h}_i$, then the propeller has enough space to move forward and backward, but possibly with different velocity bounds.

Concerning w, a lower bound $\bar{w}$ can be calculated as follows:

$$\bar{w} = \sqrt{2}L_x + 2\max\left(\sqrt{l_x^2 + \frac{d^2}{4}}, \lambda\right). \tag{31}$$

3.2. *Flying across a narrow passage*

Different from a traditional quadrotor, the proposed solution can fly across a narrow passage, with the option of reversing its direction if it detects that the path is not viable.

Since the proposed quadcopter is overactuated, in principle, it is possible to operate the rotors orientations to further reduce the cross-sectional area, allowing the vehicle to fly across narrower passages with respect to those modeled in previous sections.

The idea is to control one rotor to assume the orientation leading to the minimum possible height of the cone $\hat{h}$. Indeed, in case it holds $\|h_c\| > d$ (alternatively, $\|h_c\| \leq d$), the configuration shown in Sec. 3.1 (Sec. 3.1) is the one that minimize $\hat{h}$, i.e.

$$\hat{h} \geq \min\left(l_x + \lambda, \frac{d^2}{2\,\|h_c\|}\right). \tag{32}$$

Since this orientation may not be suitable to the force and torques required for a stable fly, the rotor can be turned off, since the remaining three propellers have to possibility to compensate the lack of the fourth rotor while this latter is crossing the opening. Afterwards, the rotor can be turned on, and the procedure can be repeated for the remaining rotors. The entire procedure can be safely completed only if the necessary space is present beyond the narrow passage. It is worth while to note that a simple strategy to ensure the controllability of the system is the use of bidirectional motor rotation: in this case, the opposite motor with respect to the one that is switched off is used to balance the system, while the other two motors carry the whole weight of the quadcopter, as shown in Fig. 11.

Another possibility could be exploit failsafe control strategies that ensure the controllability of the quadcopter despite the loss of a propeller [16]. However, this solution is more tricky, since it requires a yaw speed that cannot fit all the situations.

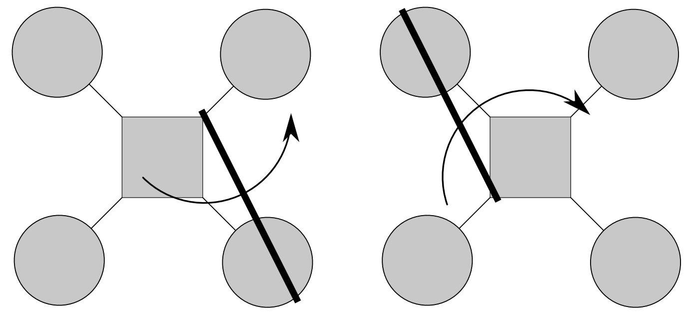

Fig. 11. Sequence of movement that allows a quadcopter to fly across a narrow passage. The black line is the minimum width of the opening should have to allow the quadcopter to pass.

The same approach can be used to deal with a window having width w. The quadrotor makes each rotor individually cross the opening, leveraging the fact that its body does not occupy the whole between the arms. As depicted in Fig. 11, for simplicity, the body of the quadrotor is modeled as a parallelepiped having width L_b. Therefore, w can be considerably less than Eq. (31), approximately equal to

$$\bar{w} = \sqrt{L_x^2 + L_b^2} + \frac{d\lambda}{\|h_c\|}. \tag{33}$$

It is worth while to note that these kind of maneuvers are also possible with traditional quadrotors. However, in the general case, there is no possibility to invert the direction in case there are obstacles behind the opening that cannot be detected before entering the opening itself. This situation may likely lead to hit the obstacle, possibly damaging the vehicle and compromising the mission.

4. Simulation Model

The physical system dynamics, including sensors and control interface, have been simulated using the OpenModelica open-source software framework.[b] The control logic itself has been implemented as a Modelica script.

By dividing the model into simple components, it is possible to easily consider effects that have been neglected in the dynamic analysis. In particular, those components mimic exactly the physical system, i.e. the body and arms of the quadcopter, the electromechanical actuators and the propellers.

The quadcopter frame has been designed to be flexible by implementing the generic formulation of the model.

[b] Open Source Modelica Consortium (OSMC), https://openmodelica.org.

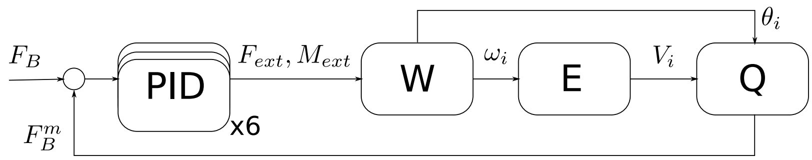

Fig. 12. Simple control scheme used during the simulations: the linear and angular errors are transformed by a set of six PID controllers in forces and torques that should be applied to the overall system **Q** to bring it to the desired position. The desired propeller speeds ω_i and orientations θ_i are computed by the wrench generation block **W**. Speed references are converted to electric voltages that are applied to the motors by the **E** block.

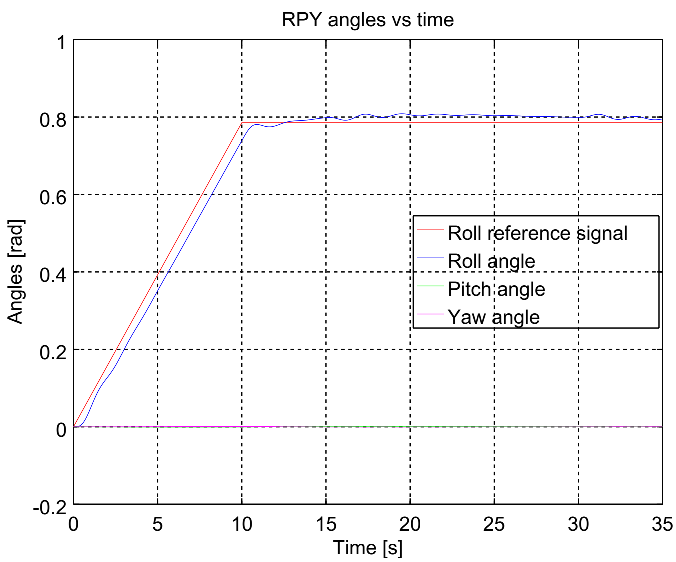

Fig. 13. Results of the simulation: roll, pitch and yaw angles. The red line is the reference signal of the roll angle and the blue line is the roll itself.

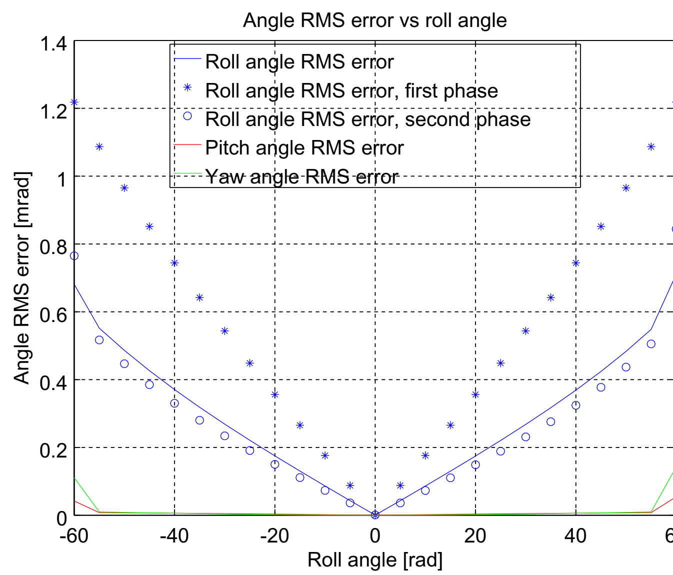

Fig. 15. RMS error on roll, pitch and yaw angles.

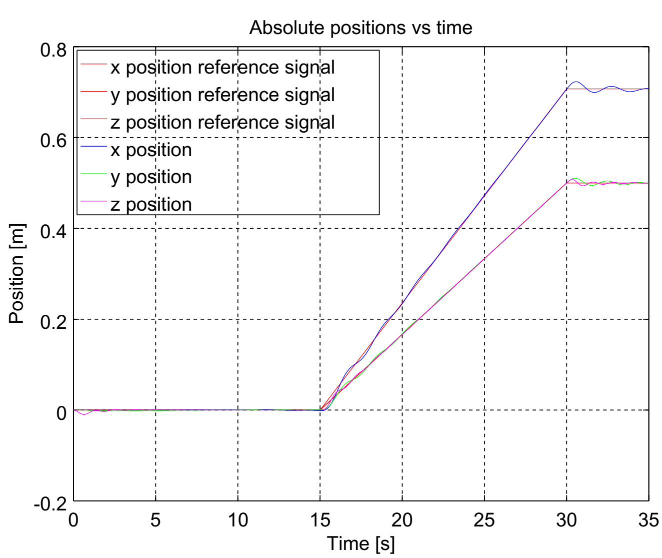

Fig. 14. Result of the simulation: x, y and z displacements and reference signals.

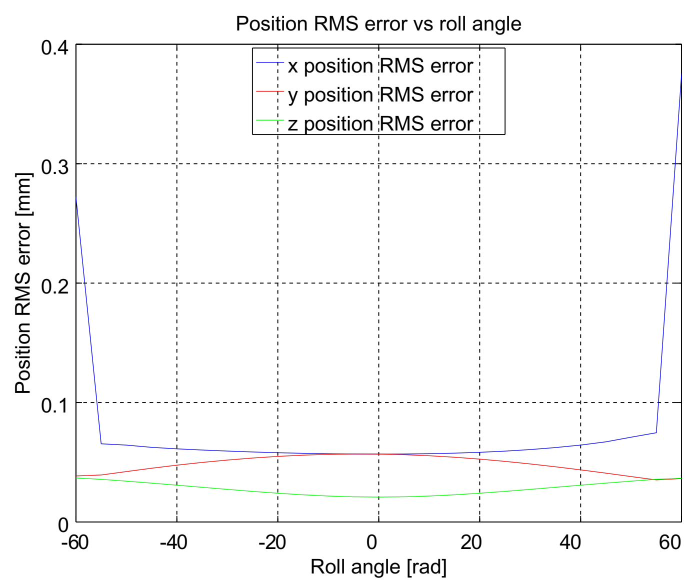

Fig. 16. RMS error on x, y and z displacements.

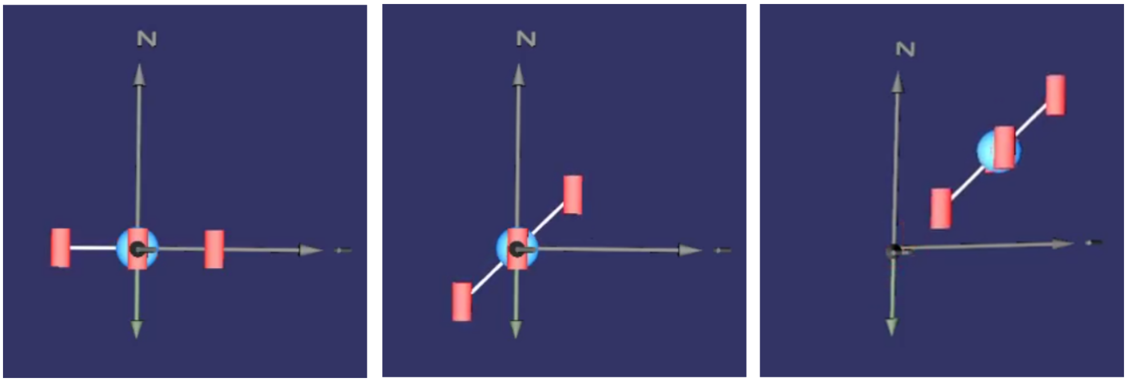

Fig. 17. The quadcopter in the different phases of the manoeuvre. In the left figure, the quadcopter is in rest in horizontal attitude, in the center one it has a 45° roll angle and in the right one it flies maintaining its attitude.

Several parameters can be set in the implemented model. The value of N can be arbitrarily changed to set the number of arms; the arms are not required to be disposed in an axial-symmetric geometry; the vectors $\mathbf{L}$ and $\mathbf{l}$ can have components along axis different from the x-direction; aerodynamic forces and torques applied to the center of gravity can be nonnull (their values can be read from a lookup table that relates quadrotor configuration and velocities).

The joints actuation has been implemented as an angular actuation with a cutoff frequency to simulate the servo actuation system which usually equips real quadcopters. Similarly, the propeller motors have been modeled as DC motors while the control variable is the input voltage. In this way, the dynamics of motors and actuators are properly simulated, despite their modeling being neglected in the analysis to simplify the dissertation.

5. Simulation Results

This section shows some simulation results carried out using the simulator described in Sec. 4. The manoeuvre consists of two steps: first the quadcopter rolls and then it follows a straight line laying on the x_B–y_B plane and directed at 45° around the z_B-axis for one meter, as described in Sec. 3.1. To control the quadcopter, a two step controller was implemented.

A set of PID controllers are fed with the error on the desired state and combined to calculate forces and torques in the 3D space required to reach the desired configuration. Afterwards, control inputs are driven to obtain the corresponding forces and torques. To implement this behavior, propellers are directed towards the force direction and, by means of a wrench generation technique, the propeller speeds are calculated. It is worth while to note that the propellers are kept almost vertical during the whole process and only slightly inclined in the desired direction during the motion. The considered manoeuvre is divided into two steps. First, the quadcopter changes its attitude and rolls. To test the behavior of the vehicle under different roll angles, an extended set of simulation was carried out. Figure 13 shows the roll angle following the reference signal while other angles remain constant.

Afterwards, the vehicle moves in the 3D space along the desired direction (Fig. 14) for 1 m. Figures 15 and 16 show the RMS error of the actual orientations with respect to the desired angular set-point. In each simulation, the roll angular set-point is set to a given value in the range [−60°, 60°]. The results show that the angle affects the quality of the control for the roll angle. The cause is that the higher the roll angle, the lower the momentum arm available for the propellers to control the roll angle. Indeed, in both phases, the roll RMS error is directly proportional to the roll angle, while in the first phase, there is also a component that derives from the higher velocity of the roll reference. Besides, the results show that the control is able to stabilize the system for roll angles $|\phi| < 60°$. Despite these results, the maximum errors, at 55° of roll angle, are about 20 mm and 60 mrad.

Figure 17 shows the 3D visualization of the manoeuvre.

6. Conclusions

This paper presented a model of a double axis tilting rotor quadcopter. The vehicle is characterized by an overactuated structure. With respect to traditional quadcopters, it has the possibility to tilt the arms and the motor and to individually orientate the thrust of each propeller.

The result is the chance to hover and translate without any significant attitude restrictions: this solution is well suited for inspecting tunnels with arbitrary orientations and narrow passages trespassing. The quadrotor has been implemented as a Modelica library that allows to model dynamics and disturbances. Simulation results demonstrate the feature of following an arbitrary path without being affected by the attitude.

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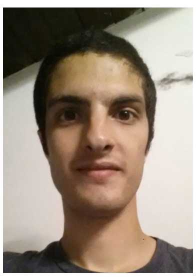

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