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Fixed-Time Adaptive Tracking Control for Multi-Agent Systems with Input Saturation and Sensor Hysteresis

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This paper considers the fixed-time control issue of a class of uncertain nonlinear multi-agent systems with external disturbances under unknown sensor hysteresis and input saturation. The hysteresis phenomenon exhibited by sensors is characterized using the Bouc–Wen model. The radial basis function neural network (RBFNN) is used to approximate the unknown nonlinear dynamics, and two adaptive rules are designed to estimate the uncertain parameters contained in the RBFNN and compensate for the sensor hysteresis, respectively. The backstepping technique is employed to design the distributed adaptive fixed-time control mechanism. The difficulty of controller design is that the exact states of the system cannot be fed back and the singularity issue in fixed-time control must be excluded. The proposed scheme ensures the practical fixed-time stability of the closed-loop system and the boundedness of all system signals. Finally, numerical simulation is conducted to confirm the validity of the theoretical results.

Keywords: Multi-agent system; sensor hysteresis; saturated control; fixed time.

1. Introduction

With the rapid advancement of computer science, the Internet, and artificial intelligence, the problem of coordinated control of multi-agent systems (MASs) has drawn the interest of a large number of scholars and has become a hot topic in the areas of control theory and artificial intelligence [1, 2]. The main issue in the coordination of MASs is the so-called consensus problem, which has a wide range of applications in multi-UAV formation control [3], multi-robot coordinated control [4], satellite formation control [5], and so on. The main idea of consensus control is that each agent coordinates its behaviors based on local information so that all agent states are consistent or synchronized [6, 7].

Nonlinear systems exist in large numbers in the real world. Due to the complexity of the real world, the system may be affected by some uncertain factors, such as uncertain nonlinear dynamics and external disturbances. To eliminate the influence of these uncertain factors, neural networks [8–11] are usually used to approximate uncertain nonlinear dynamics and to realize adaptive synchronization of systems. In practice, due to the limitations of physical devices, it is necessary to limit the system inputs to a specific range, so the development of saturation controllers is of great significance. For full state-constrained nonlinear systems with asymmetric input saturation, a command filter-based adaptive tracking control scheme was presented in [12]. In [13], the problem of consensus tracking for a class of strict-feedback uncertain nonlinear MASs with input saturation and partial state constraints was studied. For fractional-order MASs with input saturation and partial state constraints, an adaptive tracking control scheme was designed by using the event-triggered strategy through output feedback in [14].

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Recently, the fixed-time control method has attracted a large number of attention from scholars due to its rapid convergence, strong anti-interference capabilities, and good tracking performance. Unlike the finite-time control, the settling time in fixed-time control has a fixed upper bound that does not depend on the initial state [15, 16]. In practical applications, the initial states of the system may be difficult to acquire or inaccurate, which limits the practical application of the finite-time control method. Therefore, the fixed-time control approach is more applicable in practice. In [15], the fixed-time adaptive tracking control problem for uncertain nonlinear systems with unmeasurable states and input hysteresis was studied. The adaptive tracking control issue is investigated to ensure fixed-time convergence for strict-feedback systems with input hysteresis in [16].

In applications, actuators are generally used to actuate the system, while sensors are used to sense the states of the system for feedback. However, actuators and sensors in control systems often suffer from a nonlinear phenomenon known as hysteresis. Due to the effects of hysteresis, the true inputs and states of the system become unknown. It is thus evident that the stability of the system cannot be ensured until the hysteresis is compensated for. To guarantee the stability of the system is not affected by hysteresis, researchers have developed many mathematical models that describe the hysteresis phenomenon, as detailed in the existing literature [17–20]. Of these models, the modified Bouc–Wen hysteresis model in [20] is the most widely adopted within the academic community. A substantial body of research has been conducted on the control problem of uncertain nonlinear systems with hysteresis, as presented in [21–28]. In [21], the adaptive finite-time tracking control issue for multi-input and multi-output nonlinear systems with input hysteresis and unknown nonlinear dynamics was studied. In [22], the authors studied the adaptive stabilization problem of stochastic nonlinear systems with input hysteresis and unknown control directions via output feedback. The problem of event-triggered containment control for stochastic nonlinear systems with input hysteresis was investigated in [23]. In [24], the authors studied the adaptive neural control problem for nonlinear systems with output hysteresis and unmeasurable states based on output feedback. The adaptive tracking control for switched nonlinear systems with un-modeled dynamics and unknown hysteresis was studied in [25]. In [26], a finite-time adaptive control strategy was proposed for nonlinear systems with output hysteresis and disturbances. In [27], the author discussed the adaptive tracking control issue of nonlinear systems with state and actuator hysteresis. The finite-time neural adaptive control issue of nonlinear systems with state hysteresis was discussed in [28].

It can be seen from the above literature review that there have been many research works related to the control issue of systems with actuator and/or output hysteresis. However, there is very little research on the control issue of systems subject to full state hysteresis, such as those in [27, 28]. In many practical applications, it is more important to reach the desired control goals in a fixed time than to meet the asymptotic performance requirements of the control system. Therefore, it is of great theoretical and practical significance to study the fixed-time control problem of uncertain nonlinear MASs with full-state hysteresis.

Drawing inspiration from the above literature, this paper investigates the fixed-time adaptive tracking control issue for nonlinear MASs with input saturation and full-state hysteresis caused by sensors. In comparison with related works, the contributions of this paper are as follows:

(1) Taking into account the potential effects of unknown sensor hysteresis and input saturation, a fixed-time control scheme is proposed to achieve the tracking control objective in MASs with uncertain nonlinearity and external disturbances. By compensating for the sensor hysteresis in the backstepping design process, the designed control mechanism guarantees that the closed-loop system is practically fixed-time stable.

(2) Most of the existing work focuses on the situation where the actuator or system output is affected by sensor hysteresis, and this paper focuses on the situation where the full states of the system are affected by sensor hysteresis. Unlike [21–23], where the actuator hysteresis compensation is only required in the last step of the backstepping process, in this paper, the sensor hysteresis must be compensated in each step because all the states of the system are affected by the sensor hysteresis, which increases the difficulty of the algorithm design. Compared with [24–26], in which only the system output receives the effect of sensor hysteresis, this is the special case of this paper.

(3) As far as we know, some research results have been achieved in the aspect of the asymptotic stability of uncertain strict feedback nonlinear MASs with full state hysteresis [28–30]. This paper mainly focuses on the fixed-time stability of the system. Compared with the finite-time stability, the upper bound of the settling time of fixed-time stability has the advantage of being independent of the initial state. When the exact states of the system cannot be fed back, it is challenging to provide a solution for the fixed-time control of the system. Additionally, in the fixed-time control scheme proposed in this work, the virtual controllers contain only error terms with exponents greater than 1, thus avoiding the singularity problem often encountered in fixed-time control designs [31, 32].

The structure of the remaining content is as follows. For the problem statement and some preliminary knowledge, see Sec. 2. In Secs. 3 and 4, we present the adaptive control design process and carry out the stability analysis, respectively. The simulation results are given in Sec. 5. The whole paper is concluded in Sec. 6.

2. Problem Statements

2.1. *Notations*

Some explanations of mathematical symbols are given in Table 1.

Table 1. Symbol table.

R	The set of real numbers
R^+	The set of nonnegative real numbers
R^q	The set of q-dimensional real vectors
$R^{N\times N}$	The set of $N \times N$ real matrices
$\mathrm{sign}(\cdot)$	The sign function
$\mathrm{Re}(\cdot)$	The real part of a complex number
$\|\cdot\|$	The two-norm of a vector or matrix
$\underline{\sigma}(\cdot)$	The minimum singular value of a matrix
$\frac{d^q x}{dt^q}$	The qth derivative of x with respect to t
$\mathrm{diag}(b_1, \ldots, b_N)$	The diagonal matrix with diagonal elements b_1, $b_2, \ldots, b_N$
1_N	The column vector with each element being 1
$\min c_i$, $\max c_i$	The minimum and maximum values of c_i
$\arg\min_{\theta\in R^m} f(\theta)$	The value of variable θ when $f(\theta)$ is minimum

2.2. *Graph theory*

A directed graph is denoted by $\mathcal{G}(\mathcal{W}, \mathcal{S}, \mathcal{A})$ that models the interactions among agents, where $\mathcal{W} = \{1, 2, \ldots, N\}$ represents the set of nodes corresponding to N following agents, $\mathcal{S} \subseteq \mathcal{W} \times \mathcal{W}$ is the set of directed edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix. An edge $(j, i) \in \mathcal{S}$ means that agent i can obtain information from agent j. In this case, $a_{ij} > 0$, otherwise $a_{ij} = 0$. It is assumed that $a_{ii} = 0$ in this paper. The set of neighbors for agent i is defined as $\mathcal{M}_i = \{j \mid (j, i) \in \mathcal{S}\}$. Let $\mathcal{D} = \mathrm{diag}\{d_1, \ldots, d_N\}$ with $d_i = \sum_{j=1}^{N} a_{ij}$ and $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

Define another graph $\bar{\mathcal{G}}(\mathcal{W} \cup 0, \mathcal{S}, \mathcal{A})$, where the leader agent is labeled as 0. The leader agent is assumed to have no neighbors. If agent i cannot sense the leader, then, $b_i = 0$ and $b_i > 0$, otherwise. Let $\mathcal{B} = \mathrm{diag}(b_1, \ldots, b_N)$. We say that the graph $\bar{\mathcal{G}}$ has a directed spanning tree (DST) if there exists at least one directed path from the root node to every other node.

Figure 1 shows a digraph containing a DST for four followers and a leader. It is obvious that $a_{ij} = 0$ except for $a_{21} = a_{42} = a_{43} = 1, d_1 = d_3 = 0, d_2 = 1,$ and $d_4 = 2$. Thus,

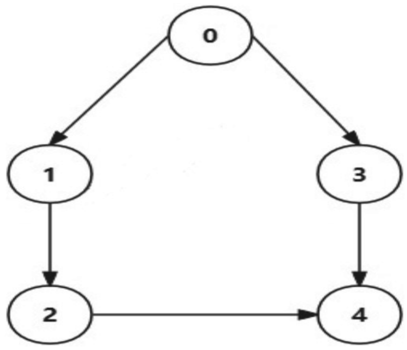

Fig. 1. A digraph having a DST.

$$\mathcal{D} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathcal{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}, \quad \text{and}$$

$$\mathcal{L} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 2 \end{pmatrix}.$$

2.3. *Problem description*

Consider an MAS consisting of N following agents, where the dynamics of the ith agent are

$$\begin{cases} \dot{x}_{i,q} = x_{i,q+1} + f_{i,q}(\bar{x}_{iq}) \\ \qquad + d_{iq}(t), \quad q = 1, \ldots, n-1, \\ \dot{x}_{i,n} = \mathrm{sat}(u_i(t)) + f_{i,n}(\bar{x}_{in}) + d_{in}(t), \\ y_i = x_{i1}, \quad i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $\bar{x}_{iq} = [x_{i1}, x_{i2}, \ldots, x_{iq}]^T$, $q = 1, 2, \ldots, n$, and let $x_i = \bar{x}_{in}$ be the full state vector. $f_{iq}(\bar{x}_{iq})$ is the uncertain nonlinear dynamics, $d_{iq}(t) \in \mathbb{R}$ is the external disturbance, y_i is the system output, and $\mathrm{sat}_i(u_i)$ is the saturation controller with u_i being its input.

The asymmetric saturated controller $\mathrm{sat}_i(u_i(t))$ is defined as

$$\mathrm{sat}_i(u_i) = \chi_i(u_i)u_i, \tag{2}$$

where

$$\chi_i(u_i) = \begin{cases} \dfrac{u_{iM}}{u_i} & \text{if} \quad u_i \geq u_{iM}, \\ 1 & \text{if} \ -u_{im} < u_i < u_{iM}, \\ -\dfrac{u_{im}}{u_i} & \text{if} \quad u_i \leq -u_{im}, \end{cases} \tag{3}$$

with u_{iM} and u_{im} being the saturation bounds, which are known positive constants. It is obvious that the function $\chi_i(u_i) \in (0,1]$, which indicates the saturation degree of u_i. When $\chi_i(u_i) = 1$, it means that no saturation occurs. For practical applications, the input u_i of the saturation controller is required to be bounded. Thus,

$$0 < o_i \leq \chi_i(u_i) \leq 1, \tag{4}$$

where the unknown constant o_i will be estimated later adaptively.

The system states $x_{iq}, q = 1, \ldots, n$, are measured by sensors with hysteresis. The Bouc–Wen hysteresis model is utilized as follows:

$$\begin{aligned} \hat{x}_{iq} &= H_{iq}(x_{iq}) \\ &= \zeta_{iq} x_{iq} + \lambda_{iq} r_{iq}, \quad q = 1, \ldots, n, \end{aligned} \tag{5}$$

where $\hat{x}_{iq}$ is the measured value of x_{iq}, ζ_{iq}, and λ_{iq} are unknown nonzero constants with the same signs, and r_{iq} is the solution of the following equation:

$$\dot{r}_{iq} = \dot{x}_{iq} g_{iq}(r_{iq}, \dot{x}_{iq}), \quad r_{iq}(t_0) = 0 \tag{6}$$

and

$$\begin{aligned} g_{iq}(r_{iq}, \dot{x}_{iq}) = 1 &- \mathrm{sign}(\dot{x}_{iq}) h_{iq}^{(1)} |r_{iq}|^{h_{iq}^{(2)}-1} r_{iq} \\ &- h_{iq}^{(3)} |r_{iq}|^{h_{iq}^{(2)}}, \end{aligned} \tag{7}$$

where $h_{iq}^{(1)}, h_{iq}^{(2)}$, and $h_{iq}^{(3)}$ are constants satisfying $h_{iq}^{(1)} > |h_{iq}^{(3)}|$, $h_{iq}^{(2)} \geq 1$, and $|r_{iq}(t)| \leq \bar{r}_{iq} = (h_{iq}^{(1)} + h_{iq}^{(3)})^{-\frac{1}{h_{iq}^{(2)}}}$. $h_{iq}^{(1)}, h_{iq}^{(3)}$, and $h_{iq}^{(2)}$ mean the shape, the amplitude and the smoothness of the hysteresis, respectively. Without loss of generality, it is assumed that $\mathrm{sign}(\zeta_{iq}) = \mathrm{sign}(\lambda_{iq}) = 1$.

The aim of this work is to design a control scheme such that the closed-loop system is practically fixed-time stable and all the system signals are bounded.

In the following, some required lemmas and assumptions are given.

Assumption 2.1. The external disturbances d_{iq}, $q = 1, \ldots, n$, satisfy $|d_{iq}| \leq \bar{d}_{iq}$ with $\bar{d}_{iq}$ being constants.

Assumption 2.2. For leader signal $y_0(t)$, $\frac{d^q y_0}{dt^q}$, $q = 1, 2, \ldots, n$, are continuous and bounded.

Assumption 2.3. Graph $\bar{\mathcal{G}}$ has a DST rooted at the leader.

Remark 2.4. Assumptions 2.1–2.3 are common assumptions in the tracking control of MASs. Similar assumptions can be found in [13, 27, 33].

Remark 2.5. Assumption 2.3 shows that $d_i + b_i \neq 0$, thus ensuring the effectiveness of the subsequently proposed controller and further guaranteeing the stability of the system.

Lemma 2.6 [33]. *For a system $\dot{x}(t) = g(t)$, if the Lyapunov function $V(x)$ satisfies*

$$\dot{V}(x) \leq -\rho_1 V^{\beta_1} - \rho_2 V^{\beta_2} + M, \tag{8}$$

where ρ_1, ρ_2, $M > 0$, $\beta_1 \in (1, \infty)$, and $\beta_2 \in (0,1)$ are constants, then the system is practically fixed-time stable, the residual set is

$$\Omega = \left\{ x \mid V \leq \min\left(\left[\frac{M}{\rho_1(1-\theta)} \right]^{\frac{1}{\beta_1}}, \left[\frac{M}{\rho_2(1-\theta)} \right]^{\frac{1}{\beta_2}} \right) \right\}, \tag{9}$$

with $\theta \in (0,1)$ being a constant, and the settling time T satisfies $T \leq T_{\max} = \frac{1}{\rho_1 \theta(\beta_1 - 1)} + \frac{1}{\rho_2 \theta(1-\beta_2)}$.

Lemma 2.7 [33]. *For $a, b \in R$ and $c_1, c_2, c_3, H \in R^+$,*

$$\begin{aligned} H|a|^{c_1}|b|^{c_2} \leq \frac{c_3 c_1}{c_1 + c_2} |a|^{c_1+c_2} \\ + \frac{c_2 H^{\frac{c_1+c_2}{c_2}}}{c_1 + c_2} c_3^{\frac{-c_1}{c_2}} |b|^{c_1+c_2}. \end{aligned} \tag{10}$$

Lemma 2.8 [34]. *For $a_i \geq 0, i = 1, \ldots, n$, one gets*

$$\begin{cases} \left(\displaystyle\sum_{i=1}^n a_i \right)^s \leq \displaystyle\sum_{i=1}^n a_i^s, & 0 < s < 1, \\ \left(\displaystyle\sum_{i=1}^n a_i \right)^s \leq n^{s-1} \left(\displaystyle\sum_{i=1}^n a_i^s \right), & s > 1. \end{cases} \tag{11}$$

Lemma 2.9 [35]. *Under Assumption 2.3, for any eigenvalue λ of $\mathcal{L} + \mathcal{B}$, $\mathrm{Re}(\lambda) > 0$ holds.*

Lemma 2.10 [36]. *For $\forall\, a, b \geq 0$, q_1 and $q_2 > 1$ satisfying $\frac{1}{q_1} + \frac{1}{q_2} = 1$, one has*

$$ab \leq \frac{a^{q_1}}{q_1} + \frac{b^{q_2}}{q_2}. \tag{12}$$

Lemma 2.11 [37]. *Let $\tilde{\theta} = \theta - \hat{\theta}$, where θ and $\hat{\theta}$ are two scalar functions. For any $0 < c = c_1/c_2 \leq 1$ with c_1 and c_2 being odd numbers, one has*

$$\tilde{\theta}\hat{\theta}^c \leq s_1 \theta^{1+c} - s_2 \tilde{\theta}^{1+c}, \tag{13}$$

where $s_1 = \frac{1}{1+c}[1 - 2^{c-1} + \frac{1}{1+c} + \frac{2^c(1-c^2)}{1+c}]$ and $s_2 = \frac{2^{c-1}}{1+c}[1 - 2^{c(c-1)}]$ are constants.

Lemma 2.12 [37]. *Let $p > 1$ be an odd number, $y > 0$ be a constant, and $x < y$. The following inequality:*

$$x(y-x) \le y^{p+1} - x^{p+1} \tag{14}$$

holds.

Lemma 2.13 [38]. *Consider system $\dot{\hat{\theta}}(t) = -m_1\hat{\theta}^{2\beta_1-1}(t) - m_2\hat{\theta}^{2\beta_2-1}(t) + s(t)$, where $\hat{\theta}(t) \in \mathbb{R}$, m_1, and $m_2 > 0$ are constants, $\beta_1 = \frac{p_1}{q_1} >, 10.5 < \beta_2 = \frac{q_2}{p_2} < 1$ with p_1, q_1, p_2, and $q_2 > 0$ being odd numbers, and $s(t)$ is a positive function. It is true that $\hat{\theta}(t) \ge 0$ for $\forall\, t \ge t_0$ if $\hat{\theta}(t_0) \ge 0$.*

2.4. *The radial basis function neural network*

According to [39], a continuous function $f(X)$ on some compact set Ω_X is usually approximated by a radial basis function neural network (RBFNN), that is

$$f(X) = \theta^{*T}\varphi(X) + \varepsilon, \tag{15}$$

where the approximation error ε satisfies $|\varepsilon| \le \bar{\varepsilon}$ with $\bar{\varepsilon} > 0$ being a constant. $X = [X_1, \dots, X_r]^T$ is the input vector of the RBFNN and $\varphi(X) = [\varphi_1(X), \dots, \varphi_m(X)]^T$ is the basis function vector with m being the nodes number of the RBFNN. $\theta^* \in R^m$ is the ideal weight vector satisfying

$$\theta^* = \arg\min_{\theta\in R^m}\left\{\sup_{X\in\Omega_X}|f(X) - \theta^T\varphi(X)|\right\} \tag{16}$$

with $\theta = [\theta_1, \theta_2, \dots, \theta_m]^T$ being the weight vector.

The basis function $\varphi_i(X)$ usually adopts the Gaussian function defined as follows:

$$\varphi_i(X) = \exp\left[-\frac{(X-\iota_i)^T(X-\iota_i)}{\upsilon_i^2}\right], \tag{17}$$

where ι_i and υ_i, $i = 1, \dots, m$, are the center and the width of the Gaussian function, respectively.

Lemma 2.14 [28]. *Let $\bar{X}_r = [X_1, \dots, X_r]$ and $\varphi(\bar{X}_r) = [\varphi_1(\bar{X}_r), \dots, \varphi_m(\bar{X}_r)]^T$ be the input vector and the basic function vector of the RBFNN, respectively. Then,*

$$\|\varphi(\bar{X}_r)\| \le \|\varphi(\bar{X}_\rho)\|, \tag{18}$$

holds for any positive integer $\rho \le r$.

3. Adaptive Control Design

Let

$$\begin{cases} z_{i1} = \Sigma_{j=1}^N a_{ij}(\hat{x}_{i1} - \hat{x}_{j1}) + b_i(\hat{x}_{i1} - y_0), \\ z_{iq} = \hat{x}_{iq} - \alpha_{i,q-1}, \end{cases} \tag{19}$$

where z_{i1} is the local tracking error, z_{iq}, $q = 2, \dots, n$, denotes the error variables between $\hat{x}_{iq}$ and the virtual controller $\alpha_{i,q-1}$.

Let $z_1 = [z_{11}, z_{21}, \dots, z_{N1}]^T$ and $\hat{y} = [\hat{y}_1, \hat{y}_2, \dots, \hat{y}_N]^T = [\hat{x}_{11}, \hat{x}_{21}, \dots, \hat{x}_{N1}]^T$. Thus,

$$z_1 = (\mathcal{L} + \mathcal{B})(\hat{y} - 1_N y_0). \tag{20}$$

Step 1. From (1) and (19), one has

$$\begin{aligned} \dot{z}_{i1} &= \sum_{j=1}^N a_{ij}(\dot{\hat{x}}_{i1} - \dot{\hat{x}}_{j1}) + b_i(\dot{\hat{x}}_{i1} - \dot{y}_0) \\ &= \sum_{j=1}^N a_{ij}\dot{\hat{x}}_{i1} - \sum_{j=1}^N a_{ij}\dot{\hat{x}}_{j1} + b_i\dot{\hat{x}}_{i1} - b_i\dot{y}_0 \\ &= (d_i + b_i)\dot{\hat{x}}_{i1} - \sum_{j=1}^N a_{ij}\dot{\hat{x}}_{j1} - b_i\dot{y}_0. \end{aligned} \tag{21}$$

Further, it follows from (1) and (5)–(7) that

$$\begin{aligned} \dot{z}_{i1} = {} & (d_i + b_i)(\zeta_{i1}\dot{x}_{i1} + \lambda_{i1}\dot{r}_{i1}) - \sum_{j=1}^N a_{ij}(\zeta_{j1}\dot{x}_{j1} \\ & + \lambda_{j1}\dot{r}_{j1}) - b_i\dot{y}_0 \\ = {} & (d_i + b_i)(\zeta_{i1} + \lambda_{i1}g_{i1})\dot{x}_{i1} - \sum_{j=1}^N a_{ij}(\zeta_{j1} \\ & + \lambda_{j1}g_{j1})\dot{x}_{j1} - b_i\dot{y}_0 \\ = {} & (d_i + b_i)(\zeta_{i1} + \lambda_{i1}g_{i1})\left(\frac{\hat{x}_{i2} - \lambda_{i2}r_{i2}}{\zeta_{i2}} + f_{i1}\right. \\ & \left. + d_{i1}\right) - \sum_{j=1}^N a_{ij}(\zeta_{j1} + \lambda_{j1}g_{j1})\left(\frac{\hat{x}_{j2} - \lambda_{j2}r_{j2}}{\zeta_{j2}}\right. \\ & \left. + f_{j1} + d_{j1}\right) - b_i\dot{y}_0 \\ = {} & (d_i + b_i)Y_{i1}(\hat{x}_{i2} - \lambda_{i2}r_{i2} + \zeta_{i2}f_{i1} + \zeta_{i2}d_{i1}) \\ & - \sum_{j=1}^N a_{ij}Y_{j1}(\hat{x}_{j2} - \lambda_{j2}r_{j2} + \zeta_{j2}f_{j1} \\ & + \zeta_{j2}d_{j1}) - b_i\dot{y}_0 \\ = {} & (d_i + b_i)Y_{i1}(z_{i2} + \alpha_{i1} - \lambda_{i2}r_{i2} \\ & + \zeta_{i2}f_{i1} + \zeta_{i2}d_{i1}) \\ & - \sum_{j=1}^N a_{ij}Y_{j1}\hat{x}_{j2} + \sum_{j=1}^N a_{ij}Y_{j1}\lambda_{j2}r_{j2} \\ & - \sum_{j=1}^N a_{ij}Y_{j1}\zeta_{j2}f_{j1} - \sum_{j=1}^N a_{ij}Y_{j1}\zeta_{j2}d_{j1} - b_i\dot{y}_0, \end{aligned} \tag{22}$$

where $d_i = \Sigma_{j=1}^{N} a_{ij}, Y_{iq} = \frac{\zeta_{iq}+\lambda_{iq}g_{iq}}{\zeta_{i,q+1}}$, $q = 1, 2, \ldots, n-1$.

From the definition of $g_{iq}(r_{iq}, \dot{x}_{iq})$ and noticing that $|r_{iq}(t)| \leq \bar{r}_{iq} = (h_{iq}^{(1)} + h_{iq}^{(3)})^{-\frac{1}{h_{iq}^{(2)}}}$, one gets

$$g_{iq}(r_{iq}, \dot{x}_{iq}) \geq 1 - (h_{iq}^{(1)} + h_{iq}^{(3)})|r_{iq}|^{h_{iq}(2)}$$
$$\geq 1 - (h_{iq}^{(1)} + h_{iq}^{(3)})\bar{r}_{iq}^{h_{iq}(2)} = 0 \quad (23)$$

and

$$g_{iq}(r_{iq}, \dot{x}_{iq}) \leq 1 + h_{iq}^{(1)}|r_{iq}|^{h_{iq}^{(2)}} + |h_{iq}^{(3)}||r_{iq}|^{h_{iq}^{(2)}}$$
$$\leq 1 + \frac{h_{iq}^{(1)} + |h_{iq}^{(3)}|}{h_{iq}^{(1)} + h_{iq}^{(3)}} \triangleq \bar{a}. \quad (24)$$

From (23) and (24), one has $0 \leq g_{iq}(r_{iq}, \dot{x}_{iq}) \leq \bar{a}$. Therefore,

$$\frac{\zeta_{iq}}{\zeta_{i,q+1}} \leq Y_{iq} \leq \frac{\zeta_{iq} + \lambda_{iq}\bar{a}}{\zeta_{i,q+1}}. \quad (25)$$

Let $\underline{Y}_{iq} = \frac{\zeta_{iq}}{\zeta_{i,q+1}}$ and $\overline{Y}_{iq} = \frac{\zeta_{iq}+\lambda_{iq}\bar{a}}{\zeta_{i,q+1}}$ be the upper and lower bounds of Y_{iq}, respectively, one has $\underline{Y}_{iq} \leq Y_{iq} \leq \overline{Y}_{iq}$. Since the upper and lower bounds of Y_{iq} are unknown, they will be estimated using adaptive rules later.

Choose the Lyapunov function V_{i1} as

$$V_{i1} = \frac{1}{2}z_{i1}^2 + \frac{1}{2\eta_{i1}}\tilde{\theta}_{i1}^2 + \frac{\underline{Y}_{i1}}{2}\tilde{\sigma}_{i1}^2, \quad (26)$$

where $\tilde{\theta}_{i1} = \theta_{i1} - \hat{\theta}_{i1}$ with $\hat{\theta}_{i1}$ being the estimate of θ_{i1}, which will be given later, and $\eta_{i1} > 0$ is a constant. $\tilde{\sigma}_{i1} = \sigma_{i1} - \hat{\sigma}_{i1}$ with $\sigma_{i1} = \frac{1}{\underline{Y}_{i1}}$ and $\hat{\sigma}_{i1} = \frac{1}{\hat{\underline{Y}}_{i1}}$, in which $\hat{\sigma}_{i1}$ and $\hat{\underline{Y}}_{i1}$ are the estimations of σ_{i1} and $\underline{Y}_{i1}$, respectively.

Calculating the time derivative of V_{i1} yields that

$$\dot{V}_{i1} = z_{i1}\dot{z}_{i1} - \frac{1}{\eta_{i1}}\tilde{\theta}_{i1}\dot{\hat{\theta}}_{i1} - \underline{Y}_{i1}\tilde{\sigma}_{i1}\dot{\hat{\sigma}}_{i1}$$
$$= z_{i1}\Bigg[(d_i + b_i)Y_{i1}(z_{i2} + \alpha_{i1} - \lambda_{i2}r_{i2} + \zeta_{i2}f_{i1} + \zeta_{i2}d_{i1}) - \sum_{j=1}^{N} a_{ij}Y_{j1}\hat{x}_{j2} + \sum_{j=1}^{N} a_{ij}Y_{j1}\lambda_{j2}r_{j2} - \sum_{j=1}^{N} a_{ij}Y_{j1}\zeta_{j2}f_{j1} - \sum_{j=1}^{N} a_{ij}Y_{j1}\zeta_{j2}d_{j1} - b_i\dot{y}_0\Bigg]$$
$$- \frac{1}{\eta_{i1}}\tilde{\theta}_{i1}\dot{\hat{\theta}}_{i1} - \underline{Y}_{i1}\tilde{\sigma}_{i1}\dot{\hat{\sigma}}_{i1}. \quad (27)$$

Applying Young's inequality yields that

$$(d_i + b_i)Y_{i1}z_{i1}z_{i2} \leq \frac{1}{2}(d_i + b_i)^2Y_{i1}^2z_{i1}^2 + \frac{1}{2}z_{i2}^2, \quad (28)$$

$$(d_i + b_i)Y_{i1}z_{i1}\zeta_{i2}d_{i1} \leq \frac{1}{2}(d_i + b_i)^2Y_{i1}^2\zeta_{i2}^2z_{i1}^2 + \frac{1}{2}\bar{d}_{i1}^2, \quad (29)$$

and

$$-z_{i1}\sum_{j=1}^{N} a_{ij}Y_{j1}\zeta_{j2}d_{j1} = -z_{i1}\sum_{j=1}^{N}(\sqrt{a_{ij}}Y_{j1}\zeta_{j2})(\sqrt{a_{ij}}d_{j1}) \leq \frac{1}{2}z_{i1}^2\sum_{j=1}^{N} a_{ij}Y_{j1}^2\zeta_{j2}^2 + \frac{1}{2}\sum_{j=1}^{N} a_{ij}\bar{d}_{j1}^2. \quad (30)$$

Putting (28)–(30) into (27) leads to

$$\dot{V}_{i1} \leq \frac{1}{2}(d_i + b_i)^2Y_{i1}^2z_{i1}^2 + \frac{1}{2}(d_i + b_i)^2Y_{i1}^2\zeta_{i2}^2z_{i1}^2 + \frac{1}{2}z_{i1}^2 - \frac{1}{2}z_{i1}^2 + \frac{1}{2}z_{i2}^2 + z_{i1}\Bigg[(d_i + b_i)Y_{i1}(\alpha_{i1} - \lambda_{i2}r_{i2} + \zeta_{i2}f_{i1}) - \sum_{j=1}^{N} a_{ij}Y_{j1}(\hat{x}_{j2} - \lambda_{j2}r_{j2} + \zeta_{j2}f_{j1} - \frac{z_{i1}}{2}Y_{j1}\zeta_{j2}^2) - b_i\dot{y}_0\Bigg] + \frac{1}{2}\bar{d}_{i1}^2 + \frac{1}{2}\sum_{j=1}^{N} a_{ij}\bar{d}_{j1}^2$$
$$- \frac{1}{\eta_{i1}}\tilde{\theta}_{i1}\dot{\hat{\theta}}_{i1} - \underline{Y}_{i1}\tilde{\sigma}_{i1}\dot{\hat{\sigma}}_{i1}. \quad (31)$$

Let $P_{i1}(\bar{Z}_{i1}) = \frac{1}{2}(d_i + b_i)^2Y_{i1}^2z_{i1} + \frac{1}{2}(d_i + b_i)^2Y_{i1}^2\zeta_{i2}^2z_{i1} + \frac{1}{2}z_{i1} + (d_i + b_i)Y_{i1}(\zeta_{i2}f_{i1} - \lambda_{i2}r_{i2}) - \sum_{j=1}^{N} a_{ij}Y_{j1}\hat{x}_{j2} + \sum_{j=1}^{N} a_{ij}Y_{j1}\lambda_{j2}r_{j2} - \sum_{j=1}^{N} a_{ij}Y_{j1}\zeta_{j2}f_{j1} + \frac{z_{i1}}{2}\sum_{j=1}^{N} a_{ij}Y_{j1}^2\zeta_{j2}^2 - b_i\dot{y}_0$, where $\bar{Z}_{i1} = [\hat{x}_{i1}, \hat{x}_{i2}, \hat{x}_{j1}, \hat{x}_{j2}, r_{i1}, r_{i2}, r_{j1}, r_{j2}, \dot{y}_0]^T$ and let $Z_{i1} = [\hat{x}_{i1}, \hat{x}_{j1}, \dot{y}_0]^T$.

Then,

$$\dot{V}_{i1} \leq -\frac{1}{2}z_{i1}^2 + \frac{1}{2}z_{i2}^2 + z_{i1}[(d_i + b_i)Y_{i1}\alpha_{i1} + P_{i1}(\bar{Z}_{i1})]$$

$$+\frac{1}{2}\bar{d}_{i1}^2+\frac{1}{2}\sum_{j=1}^{N}a_{ij}\bar{d}_{j1}^2-\frac{1}{\eta_{i1}}\tilde{\theta}_{i1}\dot{\hat{\theta}}_{i1}$$
$$-\underline{Y}_{i1}\tilde{\sigma}_{i1}\dot{\hat{\sigma}}_{i1}. \tag{32}$$

The function $P_{i1}(\bar{Z}_{i1})$ is approximated by a RBFNN $W_{i1}^T\varphi_{i1}(\bar{Z}_{i1})$ as

$$P_{i1}(\bar{Z}_{i1})=W_{i1}^T\varphi_{i1}(\bar{Z}_{i1})+\varepsilon_{i1}, \tag{33}$$

where the approximation error ε_{i1} satisfies $|\varepsilon_{i1}|\le\bar{\varepsilon}_{i1}$ and $\bar{\varepsilon}_{i1}>0$ is a constant.

By using Lemmas 2.10 and 2.14, one has

$$\begin{aligned}z_{i1}P_{i1}(\bar{Z}_{i1})&=z_{i1}W_{i1}^T\varphi_{i1}(\bar{Z}_{i1})+z_{i1}\varepsilon_{i1}\\&\le\frac{1}{2\mu_{i1}}z_{i1}^2\theta_{i1}\varphi_{i1}^T(Z_{i1})\varphi_{i1}(Z_{i1})\\&\quad+\frac{1}{2}\mu_{i1}+\frac{1}{2}z_{i1}^2+\frac{1}{2}\bar{\varepsilon}_{i1}^2,\end{aligned} \tag{34}$$

where $\theta_{i1}=\|W_{i1}\|^2$.

Substituting (34) into (32) yields that

$$\begin{aligned}\dot{V}_{i1}\le&\frac{1}{2}z_{i2}^2+z_{i1}(d_i+b_i)Y_{i1}\alpha_{i1}\\&+\frac{1}{2\mu_{i1}}z_{i1}^2\theta_{i1}\varphi_{i1}^T(Z_{i1})\varphi_{i1}(Z_{i1})\\&+\frac{1}{2}\mu_{i1}+\frac{1}{2}\bar{\varepsilon}_{i1}^2+\frac{1}{2}\bar{d}_{i1}^2+\frac{1}{2}\sum_{j=1}^{N}a_{ij}\bar{d}_{j1}^2\\&-\frac{1}{\eta_{i1}}\tilde{\theta}_{i1}\dot{\hat{\theta}}_{i1}-\underline{Y}_{i1}\tilde{\sigma}_{i1}\dot{\hat{\sigma}}_{i1}.\end{aligned} \tag{35}$$

Select the virtual controller α_{i1} and the adaptive rules $\dot{\hat{\theta}}_{i1}$, $\dot{\hat{\sigma}}_{i1}$ as

$$\alpha_{i1}=\hat{\sigma}_{i1}\hat{\alpha}_{i1}, \tag{36}$$

$$\begin{aligned}\hat{\alpha}_{i1}=\frac{1}{d_i+b_i}\bigg[&-k_{i1}z_{i1}^{2\beta_1-1}\\&-\frac{1}{2\mu_{i1}}z_{i1}\hat{\theta}_{i1}\varphi_{i1}^T(Z_{i1})\varphi_{i1}(Z_{i1})\bigg],\end{aligned} \tag{37}$$

$$\begin{aligned}\dot{\hat{\theta}}_{i1}=&-\gamma_{i1}^{(1)}\hat{\theta}_{i1}^{2\beta_1-1}-\gamma_{i1}^{(2)}\hat{\theta}_{i1}^{2\beta_2-1}\\&+\frac{\eta_{i1}}{2\mu_{i1}}z_{i1}^2\varphi_{i1}^T(Z_{i1})\varphi_{i1}(Z_{i1}),\end{aligned} \tag{38}$$

and

$$\begin{aligned}\dot{\hat{\sigma}}_{i1}=&-m_{i1}^{(1)}\hat{\sigma}_{i1}^{2\beta_1-1}-m_{i1}^{(2)}\hat{\sigma}_{i1}^{2\beta_2-1}\\&-z_{i1}(d_i+b_i)\hat{\alpha}_{i1},\end{aligned} \tag{39}$$

where $k_{i1},\gamma_{i1}^{(1)},\gamma_{i1}^{(2)},m_{i1}^{(1)},m_{i1}^{(2)}>0$, $\beta_1>1$, and $0<\beta_2<1$ are design parameters.

Remark 3.1. Since $\beta_1>1$, the virtual controller in this paper only contains error terms with exponent greater than 1, which avoids the singularity problem often encountered in fixed-time control design.

Noting that $z_{i1}\hat{\alpha}_{i1}\le 0$ and $\hat{\sigma}_{i1}\ge 0$, one has

$$\begin{aligned}z_{i1}(d_i+b_i)Y_{i1}\alpha_{i1}&\le z_{i1}(d_i+b_i)\underline{Y}_{i1}\hat{\sigma}_{i1}\hat{\alpha}_{i1}\\&=z_{i1}(d_i+b_i)\underline{Y}_{i1}\left(\frac{1}{\underline{Y}_{i1}}-\tilde{\sigma}_{i1}\right)\hat{\alpha}_{i1}\\&=z_{i1}(d_i+b_i)\hat{\alpha}_{i1}\\&\quad-z_{i1}(d_i+b_i)\underline{Y}_{i1}\tilde{\sigma}_{i1}\hat{\alpha}_{i1}.\end{aligned} \tag{40}$$

From(36)–(40), one derives that

$$\begin{aligned}\dot{V}_{i1}\le&-k_{i1}z_{i1}^{2\beta_1}+\frac{1}{2}z_{i2}^2+\frac{\gamma_{i1}^{(1)}}{\eta_{i1}}\tilde{\theta}_{i1}\hat{\theta}_{i1}^{2\beta_1-1}\\&+\frac{\gamma_{i1}^{(2)}}{\eta_{i1}}\tilde{\theta}_{i1}\hat{\theta}_{i1}^{2\beta_2-1}+m_{i1}^{(1)}\underline{Y}_{i1}\tilde{\sigma}_{i1}\hat{\sigma}_{i1}^{2\beta_1-1}\\&+m_{i1}^{(2)}\underline{Y}_{i1}\tilde{\sigma}_{i1}\hat{\sigma}_{i1}^{2\beta_2-1}+\omega_{i1}\\=&-k_{i1}z_{i1}^{2\beta_1}+\frac{1}{2}z_{i2}^2+\frac{\gamma_{i1}^{(1)}}{\eta_{i1}}\tilde{\theta}_{i1}\hat{\theta}_{i1}^{2\beta_1-1}\\&+\frac{\gamma_{i1}^{(2)}}{\eta_{i1}}\tilde{\theta}_{i1}\hat{\theta}_{i1}^{2\beta_2-1}+\frac{m_{i1}^{(1)}}{\sigma_{i1}}\tilde{\sigma}_{i1}\hat{\sigma}_{i1}^{2\beta_1-1}\\&+\frac{m_{i1}^{(2)}}{\sigma_{i1}}\tilde{\sigma}_{i1}\hat{\sigma}_{i1}^{2\beta_2-1}+\omega_{i1},\end{aligned} \tag{41}$$

where $\omega_{i1}=\frac{1}{2}\mu_{i1}+\frac{1}{2}\bar{\varepsilon}_{i1}^2+\frac{1}{2}\bar{d}_{i1}^2+\frac{1}{2}\sum_{j=1}^{N}a_{ij}\bar{d}_{j1}^2$.

Step 2. From (19), $z_{i2}=\hat{x}_{i2}-\alpha_{i1}$, thus,

$$\begin{aligned}\dot{z}_{i2}&=\dot{\hat{x}}_{i2}-\dot{\alpha}_{i1}\\&=\zeta_{i2}\dot{x}_{i2}+\lambda_{i2}\dot{r}_{i2}-\dot{\alpha}_{i1}\\&=(\zeta_{i2}+\lambda_{i2}g_{i2})\dot{x}_{i2}-\dot{\alpha}_{i1}\\&=(\zeta_{i2}+\lambda_{i2}g_{i2})\left(\frac{\hat{x}_{i3}-\lambda_{i3}r_{i3}}{\zeta_{i3}}+f_{i2}+d_{i2}\right)-\dot{\alpha}_{i1}\\&=Y_{i2}(\hat{x}_{i3}-\lambda_{i3}r_{i3}+\zeta_{i3}f_{i2}+\zeta_{i3}d_{i2})-\dot{\alpha}_{i1}\\&=Y_{i2}(z_{i3}+\alpha_{i2}-\lambda_{i3}r_{i3}+\zeta_{i3}f_{i2}+\zeta_{i3}d_{i2})-\dot{\alpha}_{i1}.\end{aligned} \tag{42}$$

Construct the Lyapunov function V_{i2} as

$$V_{i2}=\frac{1}{2}z_{i2}^2+\frac{1}{2\eta_{i2}}\tilde{\theta}_{i2}^2+\frac{\underline{Y}_{i2}}{2}\tilde{\sigma}_{i2}^2, \tag{43}$$

where $\tilde{\theta}_{i2}=\theta_{i2}-\hat{\theta}_{i2}$ with $\hat{\theta}_{i2}$ being the estimate of θ_{i2}, which will be given later, and $\eta_{i2}>0$ is a constant. $\tilde{\sigma}_{i2}=\sigma_{i2}-\hat{\sigma}_{i2}$

with $\sigma_{i2} = \frac{1}{\underline{Y}_{i2}}$ and $\hat{\sigma}_{i2} = \frac{1}{\hat{\underline{Y}}_{i2}}$, in which $\hat{\sigma}_{i2}$ and $\hat{\underline{Y}}_{i2}$ are the estimations of σ_{i2} and $\underline{Y}_{i2}$, respectively.

Calculating the time derivative of V_{i2} yields that

$$\begin{aligned}\dot{V}_{i2} &= z_{i2}\dot{z}_{i2} - \frac{1}{\eta_{i2}}\tilde{\theta}_{i2}\dot{\hat{\theta}}_{i2} - \underline{Y}_{i2}\tilde{\sigma}_{i2}\dot{\hat{\sigma}}_{i2} \\ &= z_{i2}[Y_{i2}(z_{i3} + \alpha_{i2} - \lambda_{i3}r_{i3} \\ &\quad + \zeta_{i3}f_{i2} + \zeta_{i3}d_{i2}) - \dot{\alpha}_{i1}] \\ &\quad - \frac{1}{\eta_{i2}}\tilde{\theta}_{i2}\dot{\hat{\theta}}_{i2} - \underline{Y}_{i2}\tilde{\sigma}_{i2}\dot{\hat{\sigma}}_{i2}.\end{aligned} \tag{44}$$

Applying Young's inequality yields that

$$Y_{i2}z_{i2}z_{i3} \le \frac{1}{2}Y_{i2}^2 z_{i2}^2 + \frac{1}{2}z_{i3}^2 \tag{45}$$

and

$$Y_{i2}\zeta_{i3}z_{i2}d_{i2} \le \frac{1}{2}Y_{i2}^2\zeta_{i3}^2 z_{i2}^2 + \frac{1}{2}\bar{d}_{i2}^2. \tag{46}$$

Incorporating (45) and (46) into (44) results in

$$\begin{aligned}\dot{V}_{i2} &\le \frac{1}{2}Y_{i2}^2 z_{i2}^2 + \frac{1}{2}Y_{i2}^2\zeta_{i3}^2 z_{i2}^2 + \frac{1}{2}z_{i3}^2 + z_{i2}^2 - z_{i2}^2 \\ &\quad + z_{i2}[Y_{i2}(\alpha_{i2} - \lambda_{i3}r_{i3} + \zeta_{i3}f_{i2}) - \dot{\alpha}_{i1}] \\ &\quad + \frac{1}{2}\bar{d}_{i2}^2 - \frac{1}{\eta_{i2}}\tilde{\theta}_{i2}\dot{\hat{\theta}}_{i2} - \underline{Y}_{i2}\tilde{\sigma}_{i2}\dot{\hat{\sigma}}_{i2}.\end{aligned} \tag{47}$$

Let $P_{i2}(\bar{Z}_{i2}) = z_{i2}(\frac{1}{2}Y_{i2}^2 + \frac{1}{2}Y_{i2}^2\zeta_{i3}^2 + 1) + Y_{i2}(\zeta_{i3} f_{i2} - \lambda_{i3}r_{i3}) - \dot{\alpha}_{i1}$, where $\bar{Z}_{i2} = [\hat{x}_{i1}, \hat{x}_{i2}, \hat{\theta}_{i1}, \hat{\sigma}_{i1}, r_{i1}, r_{i2}, r_{i3}]^T$ and let $Z_{i2} = [\hat{x}_{i1}, \hat{x}_{i2}, \hat{\theta}_{i1}, \hat{\sigma}_{i1}]^T$.

Then,

$$\begin{aligned}\dot{V}_{i2} &\le z_{i2}[Y_{i2}\alpha_{i2} + P_{i2}(\bar{Z}_{i2})] + \frac{1}{2}z_{i3}^2 + \frac{1}{2}\bar{d}_{i2}^2 \\ &\quad - \frac{1}{\eta_{i2}}\tilde{\theta}_{i2}\dot{\hat{\theta}}_{i2} - \underline{Y}_{i2}\tilde{\sigma}_{i2}\dot{\hat{\sigma}}_{i2}.\end{aligned} \tag{48}$$

The function $P_{i2}(\bar{Z}_{i2})$ is approximated by a RBFNN $W_{i2}^T\varphi_{i2}(\bar{Z}_{i2})$ as

$$P_{i2}(\bar{Z}_{i2}) = W_{i2}^T\varphi_{i2}(\bar{Z}_{i2}) + \varepsilon_{i2}, \tag{49}$$

where the approximation error ε_{i2} satisfies $|\varepsilon_{i2}| \le \bar{\varepsilon}_{i2}$ and $\bar{\varepsilon}_{i2} > 0$ is a constant.

By using Lemmas 2.10 and 2.14, one has

$$\begin{aligned}z_{i2}P_{i2}(\bar{Z}_{i2}) &= z_{i2}W_{i2}^T\varphi_{i2}(\bar{Z}_{i2}) + z_{i2}\varepsilon_{i2} \\ &\le \frac{1}{2\mu_{i2}}z_{i2}^2\theta_{i2}\varphi_{i2}^T(Z_{i2})\varphi_{i2}(Z_{i2}) + \frac{1}{2}\mu_{i2} \\ &\quad + \frac{1}{2}z_{i2}^2 + \frac{1}{2}\bar{\varepsilon}_{i2}^2,\end{aligned} \tag{50}$$

where $\theta_{i2} = \|W_{i2}\|^2$.

Putting (50) into (48) leads to

$$\begin{aligned}\dot{V}_{i2} &\le \frac{1}{2}z_{i3}^2 + z_{i2}Y_{i2}\alpha_{i2} + \frac{1}{2\mu_{i2}}z_{i2}^2\theta_{i2}\varphi_{i2}^T(Z_{i2})\varphi_{i2}(Z_{i2}) \\ &\quad + \frac{1}{2}\mu_{i2} - \frac{1}{2}z_{i2}^2 + \frac{1}{2}\bar{\varepsilon}_{i2}^2 + \frac{1}{2}\bar{d}_{i2}^2 \\ &\quad - \frac{1}{\eta_{i2}}\tilde{\theta}_{i2}\dot{\hat{\theta}}_{i2} - \underline{Y}_{i2}\tilde{\sigma}_{i2}\dot{\hat{\sigma}}_{i2}.\end{aligned} \tag{51}$$

Select the virtual controller α_{i2} and the adaptive rules $\dot{\hat{\theta}}_{i2}$, $\dot{\hat{\sigma}}_{i2}$ as

$$\alpha_{i2} = \hat{\sigma}_{i2}\hat{\alpha}_{i2}, \tag{52}$$

$$\begin{aligned}\hat{\alpha}_{i2} &= -k_{i2}z_{i2}^{2\beta_1 - 1} \\ &\quad - \frac{1}{2\mu_{i2}}z_{i2}\hat{\theta}_{i2}\varphi_{i2}^T(Z_{i2})\varphi_{i2}(Z_{i2}),\end{aligned} \tag{53}$$

$$\begin{aligned}\dot{\hat{\theta}}_{i2} &= -\gamma_{i2}^{(1)}\hat{\theta}_{i2}^{2\beta_1 - 1} - \gamma_{i2}^{(2)}\hat{\theta}_{i2}^{2\beta_2 - 1} \\ &\quad + \frac{\eta_{i2}}{2\mu_{i2}}z_{i2}^2\varphi_{i2}^T(Z_{i2})\varphi_{i2}(Z_{i2}),\end{aligned} \tag{54}$$

and

$$\dot{\hat{\sigma}}_{i2} = -m_{i2}^{(1)}\hat{\sigma}_{i2}^{2\beta_1 - 1} - m_{i2}^{(2)}\hat{\sigma}_{i2}^{2\beta_2 - 1} - z_{i2}\hat{\alpha}_{i2}, \tag{55}$$

where $k_{i2}, \gamma_{i2}^{(1)}, \gamma_{i2}^{(2)}, m_{i2}^{(1)}, m_{i2}^{(2)} > 0$, $\beta_1 > 1$, and $0 < \beta_2 < 1$ are design parameters.

Noting that $z_{i2}\hat{\alpha}_{i2} \le 0$ and $\hat{\sigma}_{i2} \ge 0$, one has

$$\begin{aligned}z_{i2}Y_{i2}\alpha_{i2} &\le z_{i2}\underline{Y}_{i2}\hat{\sigma}_{i2}\hat{\alpha}_{i2} \\ &= z_{i2}\underline{Y}_{i2}\left(\frac{1}{\underline{Y}_{i2}} - \tilde{\sigma}_{i2}\right)\hat{\alpha}_{i2} \\ &= z_{i2}\hat{\alpha}_{i2} - z_{i2}\underline{Y}_{i2}\tilde{\sigma}_{i2}\hat{\alpha}_{i2}.\end{aligned} \tag{56}$$

From (52)–(56), one derives that

$$\begin{aligned}\dot{V}_{i2} &\le -k_{i2}z_{i2}^{2\beta_1} + \frac{1}{2}z_{i3}^2 - \frac{1}{2}z_{i2}^2 + \frac{\gamma_{i2}^{(1)}}{\eta_{i2}}\tilde{\theta}_{i2}\hat{\theta}_{i2}^{2\beta_1 - 1} \\ &\quad + \frac{\gamma_{i2}^{(2)}}{\eta_{i2}}\tilde{\theta}_{i2}\hat{\theta}_{i2}^{2\beta_2 - 1} + m_{i2}^{(1)}\underline{Y}_{i2}\tilde{\sigma}_{i2}\hat{\sigma}_{i2}^{2\beta_1 - 1} \\ &\quad + m_{i2}^{(2)}\underline{Y}_{i2}\tilde{\sigma}_{i2}\hat{\sigma}_{i2}^{2\beta_2 - 1} + \omega_{i2} \\ &= -k_{i2}z_{i2}^{2\beta_1} + \frac{1}{2}z_{i3}^2 - \frac{1}{2}z_{i2}^2 + \frac{\gamma_{i2}^{(1)}}{\eta_{i2}}\tilde{\theta}_{i2}\hat{\theta}_{i2}^{2\beta_1 - 1} \\ &\quad + \frac{\gamma_{i2}^{(2)}}{\eta_{i2}}\tilde{\theta}_{i2}\hat{\theta}_{i2}^{2\beta_2 - 1} + \frac{m_{i2}^{(1)}}{\sigma_{i2}}\tilde{\sigma}_{i2}\hat{\sigma}_{i2}^{2\beta_1 - 1} \\ &\quad + \frac{m_{i2}^{(2)}}{\sigma_{i2}}\tilde{\sigma}_{i2}\hat{\sigma}_{i2}^{2\beta_2 - 1} + \omega_{i2},\end{aligned} \tag{57}$$

where $\omega_{i2} = \frac{1}{2}\mu_{i2} + \frac{1}{2}\bar{\varepsilon}_{i2}^2 + \frac{1}{2}\bar{d}_{i2}^2$.

Step q ($3 \le q \le n - 1$). It follows from (19) that

$$\dot{z}_{iq} = \dot{\hat{x}}_{iq} - \dot{\alpha}_{i,q-1}$$

$$= \zeta_{iq}\dot{x}_{iq} + \lambda_{iq}\dot{r}_{iq} - \dot{\alpha}_{i,q-1}$$
$$= (\zeta_{iq} + \lambda_{iq}g_{iq})\dot{x}_{iq} - \dot{\alpha}_{i,q-1}$$
$$= (\zeta_{iq} + \lambda_{iq}g_{iq})\Bigg(\frac{\hat{x}_{i,q+1} - \lambda_{i,q+1}r_{i,q+1}}{\zeta_{i,q+1}} + f_{iq} + d_{iq}\Bigg) - \dot{\alpha}_{i,q-1}$$
$$= Y_{iq}(\hat{x}_{i,q+1} - \lambda_{i,q+1}r_{i,q+1} + \zeta_{i,q+1}f_{iq} + \zeta_{i,q+1}d_{iq}) - \dot{\alpha}_{i,q-1}$$
$$= Y_{iq}(z_{i,q+1} + \alpha_{iq} - \lambda_{i,q+1}r_{i,q+1} + \zeta_{i,q+1}f_{iq} + \zeta_{i,q+1}d_{iq}) - \dot{\alpha}_{i,q-1}. \tag{58}$$

Choose the Lyapunov function V_{iq} as

$$V_{iq} = \frac{1}{2}z_{iq}^2 + \frac{1}{2\eta_{iq}}\tilde{\theta}_{iq}^2 + \frac{\underline{Y}_{iq}}{2}\tilde{\sigma}_{iq}^2, \tag{59}$$

where $\tilde{\theta}_{iq} = \theta_{iq} - \hat{\theta}_{iq}$ with $\hat{\theta}_{iq}$ being the estimate of θ_{iq}, which will be given later, and $\eta_{iq} > 0$ is a constant. $\tilde{\sigma}_{iq} = \sigma_{iq} - \hat{\sigma}_{iq}$ with $\sigma_{iq} = \frac{1}{\underline{Y}_{iq}}$ and $\hat{\sigma}_{iq} = \frac{1}{\hat{\underline{Y}}_{iq}}$, in which $\hat{\sigma}_{i2}$ and $\hat{\underline{Y}}_{i2}$ are the estimations of σ_{i2} and $\underline{Y}_{i2}$, respectively.

Calculating the time derivative of V_{iq} yields that

$$\dot{V}_{iq} = z_{iq}\dot{z}_{iq} - \underline{Y}_{iq}\tilde{\sigma}_{iq}\dot{\hat{\sigma}}_{iq} - \frac{1}{\eta_{iq}}\tilde{\theta}_{iq}\dot{\hat{\theta}}_{iq}$$
$$= z_{iq}[Y_{iq}(z_{i,q+1} + \alpha_{iq} - \lambda_{i,q+1}r_{i,q+1} + \zeta_{i,q+1}f_{iq} + \zeta_{i,q+1}d_{iq}) - \dot{\alpha}_{i,q-1}] - \frac{1}{\eta_{iq}}\tilde{\theta}_{iq}\dot{\hat{\theta}}_{iq} - \underline{Y}_{iq}\tilde{\sigma}_{iq}\dot{\hat{\sigma}}_{iq}. \tag{60}$$

Using Young's inequality, we have

$$Y_{iq}z_{iq}z_{i,q+1} \le \frac{1}{2}Y_{iq}^2z_{iq}^2 + \frac{1}{2}z_{i,q+1}^2 \tag{61}$$

and

$$Y_{iq}\zeta_{i,q+1}z_{iq}d_{iq} \le \frac{1}{2}Y_{iq}^2\zeta_{i,q+1}^2z_{iq}^2 + \frac{1}{2}\bar{d}_{iq}^2. \tag{62}$$

Inserting (61) and (62) into (60) yields that

$$\dot{V}_{iq} \le \frac{1}{2}Y_{iq}^2z_{iq}^2 + \frac{1}{2}Y_{iq}^2\zeta_{i,q+1}^2z_{iq}^2 + \frac{1}{2}z_{i,q+1}^2 + z_{iq}^2 - z_{iq}^2 + z_{iq}[Y_{iq}(\alpha_{iq} - \lambda_{i,q+1}r_{i,q+1} + \zeta_{i,q+1}f_{iq}) - \dot{\alpha}_{i,q-1}] + \frac{1}{2}\bar{d}_{iq}^2 - \frac{1}{\eta_{iq}}\tilde{\theta}_{iq}\dot{\hat{\theta}}_{iq} - \underline{Y}_{iq}\tilde{\sigma}_{iq}\dot{\hat{\sigma}}_{iq}. \tag{63}$$

Let $P_{iq}(\bar{Z}_{iq}) = z_{iq}(\frac{1}{2}Y_{iq}^2 + \frac{1}{2}Y_{iq}^2\zeta_{i,q+1}^2 + 1) + Y_{iq}(\zeta_{i,q+1}f_{iq} - \lambda_{i,q+1}r_{i,q+1}) - \dot{\alpha}_{i,q-1}$, where $\bar{Z}_{iq} = [\hat{x}_{i1}, \hat{x}_{i2}, \dots, \hat{x}_{iq}, r_{i1}, r_{i2}, \dots, r_{i,q+1}, \hat{\theta}_{i1}, \dots, \hat{\theta}_{i,q-1}, \hat{\sigma}_{i1}, \dots, \hat{\sigma}_{i,q-1}]^T$ and let $Z_{iq} = [\hat{x}_{i1}, \hat{x}_{i2}, \dots, \hat{x}_{iq}, \hat{\theta}_{i1}, \dots, \hat{\theta}_{i,q-1}, \hat{\sigma}_{i1}, \dots, \hat{\sigma}_{i,q-1}]^T$.

Thus,

$$\dot{V}_{iq} \le z_{iq}[Y_{iq}\alpha_{i,q} + P_{iq}(\bar{Z}_{iq})] + \frac{1}{2}z_{i,q+1}^2 + \frac{1}{2}\bar{d}_{iq}^2 - \frac{1}{\eta_{iq}}\tilde{\theta}_{iq}\dot{\hat{\theta}}_{iq} - \underline{Y}_{iq}\tilde{\sigma}_{iq}\dot{\hat{\sigma}}_{iq}. \tag{64}$$

The function $P_{iq}(\bar{Z}_{iq})$ is approximated by a RBFNN $W_{iq}^T\varphi_{iq}(\bar{Z}_{iq})$ as

$$P_{iq}(\bar{Z}_{iq}) = W_{iq}^T\varphi_{iq}(\bar{Z}_{iq}) + \varepsilon_{iq}, \tag{65}$$

where the approximation error ε_{iq} satisfies $|\varepsilon_{iq}| \le \bar{\varepsilon}_{iq}$ and $\bar{\varepsilon}_{iq} > 0$ is a constant.

By using Lemmas 2.10 and 2.14, one has

$$z_{iq}P_{iq}(\bar{Z}_{iq}) = z_{iq}W_{iq}^T\varphi_{iq}(\bar{Z}_{iq}) + z_{iq}\varepsilon_{iq}$$
$$\le \frac{1}{2\mu_{iq}}z_{iq}^2\theta_{iq}\varphi_{iq}^T(Z_{iq})\varphi_{iq}(Z_{iq}) + \frac{1}{2}\mu_{iq} + \frac{1}{2}z_{iq}^2 + \frac{1}{2}\bar{\varepsilon}_{iq}^2, \tag{66}$$

where $\theta_{iq} = \|W_{iq}\|^2$.

Putting (66) into (64) leads to

$$\dot{V}_{iq} \le \frac{1}{2}z_{i,q+1}^2 + z_{iq}Y_{iq}\alpha_{iq} + \frac{1}{2\mu_{iq}}z_{iq}^2\theta_{iq}\varphi_{iq}^T(Z_{iq})\varphi_{iq}(Z_{iq}) + \frac{1}{2}\mu_{iq} - \frac{1}{2}z_{iq}^2 + \frac{1}{2}\bar{\varepsilon}_{iq}^2 + \frac{1}{2}\bar{d}_{iq}^2 - \underline{Y}_{iq}\tilde{\sigma}_{iq}\dot{\hat{\sigma}}_{iq} - \frac{1}{\eta_{iq}}\tilde{\theta}_{iq}\dot{\hat{\theta}}_{iq}. \tag{67}$$

Select the virtual controller α_{iq} and the adaptive rules $\dot{\hat{\theta}}_{iq}$, $\dot{\hat{\sigma}}_{iq}$ as

$$\alpha_{iq} = \hat{\sigma}_{iq}\hat{\alpha}_{iq}, \tag{68}$$
$$\hat{\alpha}_{iq} = -k_{iq}z_{iq}^{2\beta_1-1} - \frac{1}{2\mu_{iq}}z_{iq}\hat{\theta}_{iq}\varphi_{iq}^T(Z_{iq})\varphi_{iq}(Z_{iq}), \tag{69}$$
$$\dot{\hat{\theta}}_{iq} = -\gamma_{iq}^{(1)}\hat{\theta}_{iq}^{2\beta_1-1} - \gamma_{iq}^{(2)}\hat{\theta}_{iq}^{2\beta_2-1} + \frac{\eta_{iq}}{2\mu_{iq}}z_{iq}^2\varphi_{iq}^T(Z_{iq})\varphi_{iq}(Z_{iq}), \tag{70}$$

and

$$\dot{\hat{\sigma}}_{iq} = -z_{iq}\hat{\alpha}_{iq} - m_{iq}^{(1)}\hat{\sigma}_{iq}^{2\beta_1-1} - m_{iq}^{(2)}\hat{\sigma}_{iq}^{2\beta_2-1}, \tag{71}$$

where $k_{iq}, \gamma_{iq}^{(1)}, \gamma_{iq}^{(2)}, m_{iq}^{(1)}, m_{iq}^{(2)} > 0$, $\beta_1 > 1$, and $0 < \beta_2 < 1$ are design parameters.

Noting that $z_{iq}\hat{\alpha}_{iq} \le 0$ and $\hat{\sigma}_{iq} \ge 0$, one has

$$\begin{aligned} z_{iq}Y_{iq}\alpha_{iq} &\le z_{iq}\underline{Y}_{iq}\hat{\sigma}_{iq}\hat{\alpha}_{iq} \\ &= z_{iq}\underline{Y}_{iq}\left(\frac{1}{\underline{Y}_{iq}} - \tilde{\sigma}_{iq}\right)\hat{\alpha}_{iq} \\ &= z_{iq}\hat{\alpha}_{iq} - z_{iq}\underline{Y}_{iq}\tilde{\sigma}_{iq}\hat{\alpha}_{iq}. \end{aligned} \tag{72}$$

From (68)–(72), one gets that

$$\begin{aligned} \dot{V}_{iq} \le& -k_{iq}z_{iq}^{2\beta_1} + \frac{1}{2}z_{i,q+1}^2 - \frac{1}{2}z_{iq}^2 + \frac{\gamma_{iq}^{(1)}}{\eta_{iq}}\tilde{\theta}_{iq}\hat{\theta}_{iq}^{2\beta_1-1} \\ &+ \frac{\gamma_{iq}^{(2)}}{\eta_{iq}}\tilde{\theta}_{iq}\hat{\theta}_{iq}^{2\beta_2-1} + m_{iq}^{(1)}\underline{Y}_{iq}\tilde{\sigma}_{iq}\hat{\sigma}_{iq}^{2\beta_1-1} \\ &+ m_{iq}^{(2)}\underline{Y}_{iq}\tilde{\sigma}_{iq}\hat{\sigma}_{iq}^{2\beta_2-1} + \omega_{iq} \\ =& -k_{iq}z_{iq}^{2\beta_1} + \frac{1}{2}z_{i,q+1}^2 - \frac{1}{2}z_{iq}^2 + \frac{\gamma_{iq}^{(1)}}{\eta_{iq}}\tilde{\theta}_{iq}\hat{\theta}_{iq}^{2\beta_1-1} \\ &+ \frac{\gamma_{iq}^{(2)}}{\eta_{iq}}\tilde{\theta}_{iq}\hat{\theta}_{iq}^{2\beta_2-1} + \frac{m_{iq}^{(1)}}{\sigma_{iq}}\tilde{\sigma}_{iq}\hat{\sigma}_{iq}^{2\beta_1-1} \\ &+ \frac{m_{iq}^{(2)}}{\sigma_{iq}}\tilde{\sigma}_{iq}\hat{\sigma}_{iq}^{2\beta_2-1} + \omega_{iq}, \end{aligned} \tag{73}$$

where $\omega_{iq} = \frac{1}{2}\mu_{iq} + \frac{1}{2}\bar{\varepsilon}_{iq}^2 + \frac{1}{2}\bar{d}_{iq}^2$.

Step n. From (19), one has

$$\begin{aligned} \dot{z}_{in} &= \dot{\hat{x}}_{in} - \dot{\alpha}_{i,n-1} \\ &= \zeta_{in}\dot{x}_{in} + \lambda_{in}\dot{r}_{in} - \dot{\alpha}_{i,n-1} \\ &= (\zeta_{in} + \lambda_{in}g_{in})\dot{x}_{in} - \dot{\alpha}_{i,n-1} \\ &= (\zeta_{in} + \lambda_{in}g_{in})(\mathrm{sat}(u_i) \\ &\quad + f_{in} + d_{in}) - \dot{\alpha}_{i,n-1} \\ &= (\zeta_{in} + \lambda_{in}g_{in})(\chi_i(u_i)u_i(t) \\ &\quad + f_{in} + d_{in}) - \dot{\alpha}_{i,n-1} \\ &= Y_{in}[\chi_i(u_i)u_i(t) + f_{in} + d_{in}] - \dot{\alpha}_{i,n-1}, \end{aligned} \tag{74}$$

where $Y_{in} = \zeta_{in} + \lambda_{in}g_{in}$.

Construct the Lyapunov function candidate V_{in} as

$$V_{in} = \frac{1}{2}z_{in}^2 + \frac{1}{2\eta_{in}}\tilde{\theta}_{in}^2 + \frac{l_i}{2}\tilde{\sigma}_{in}^2, \tag{75}$$

where $\tilde{\theta}_{in} = \theta_{in} - \hat{\theta}_{in}$ with $\hat{\theta}_{in}$ being the estimate of θ_{in}, which will be given later, $\eta_{in} > 0$ is a constant, and $l_i = \underline{Y}_{in}o_i$, in which o_i is given in (4). $\tilde{\sigma}_{in} = \sigma_{in} - \hat{\sigma}_{in}$ with $\sigma_{in} = \frac{1}{l_i}$ and $\hat{\sigma}_{in} = \frac{1}{\hat{l}_i}$, where $\hat{\sigma}_{in}$ and $\hat{l}_i$ are the estimations of σ_{in} and l_i, respectively.

Calculating the time derivative of V_{in} yields that

$$\begin{aligned} \dot{V}_{in} &= z_{in}\dot{z}_{in} - \frac{1}{\eta_{in}}\tilde{\theta}_{in}\dot{\hat{\theta}}_{in} - l_i\tilde{\sigma}_{in}\dot{\hat{\sigma}}_{in} \\ &= z_{in}[Y_{in}\chi_i(u_i)u_i(t) + Y_{in}f_{in} + Y_{in}d_{in} - \dot{\alpha}_{i,n-1}] \\ &\quad + z_{in}^2 - z_{in}^2 - \frac{1}{\eta_{in}}\tilde{\theta}_{in}\dot{\hat{\theta}}_{in} - l_i\tilde{\sigma}_{in}\dot{\hat{\sigma}}_{in}. \end{aligned} \tag{76}$$

Using Young's inequality, one has

$$Y_{in}z_{in}d_{in} \le \frac{1}{2}Y_{in}^2z_{in}^2 + \frac{1}{2}\bar{d}_{in}^2. \tag{77}$$

Let $P_{in}(\bar{Z}_{in}) = Y_{in}f_{in} + \frac{1}{2}Y_{in}^2z_{in} + z_{in} - \dot{\alpha}_{i,n-1}$, where $\bar{Z}_{in} = [\hat{x}_{i1}, \dots, \hat{x}_{in}, r_{i1}, \dots, r_{in}, \hat{\theta}_{i,1}, \dots, \hat{\theta}_{i,n-1}, \hat{\sigma}_{i,1}, \dots, \hat{\sigma}_{i,n-1}]^T$ and let $Z_{in} = [\hat{x}_{i1}, \dots, \hat{x}_{in}, \hat{\theta}_{i,1}, \dots, \hat{\theta}_{i,n-1}, \hat{\sigma}_{i,1}, \dots, \hat{\sigma}_{i,n-1}]^T$.

Thus,

$$\begin{aligned} \dot{V}_{in} &= z_{in}[Y_{in}\chi_i(u_i)u_i(t) + P_{in}(\bar{Z}_{in})] - z_{in}^2 + \frac{1}{2}\bar{d}_{in}^2 \\ &\quad - \frac{1}{\eta_{in}}\tilde{\theta}_{in}\dot{\hat{\theta}}_{in} - l_i\tilde{\sigma}_{in}\dot{\hat{\sigma}}_{in}. \end{aligned} \tag{78}$$

The function $P_{in}(\bar{Z}_{in})$ is approximated by a RBFNN $W_{in}^T\varphi_{in}(\bar{Z}_{in})$ as

$$P_{in}(\bar{Z}_{in}) = W_{in}^T\varphi_{in}(\bar{Z}_{in}) + \varepsilon_{in}, \tag{79}$$

where the approximation error ε_{in} satisfies $|\varepsilon_{in}| \le \bar{\varepsilon}_{in}$ and $\bar{\varepsilon}_{in} > 0$ is a constant.

By using Lemmas 2.10 and 2.14, one has

$$\begin{aligned} z_{in}P_{in}(\bar{Z}_{in}) &= z_{in}W_{in}^T\varphi_{in}(\bar{Z}_{in}) + z_{in}\varepsilon_{in} \\ &\le \frac{1}{2\mu_{in}}z_{in}^2\theta_{in}\varphi_{in}^T(Z_{in})\varphi_{in}(Z_{in}) + \frac{1}{2}\mu_{in} \\ &\quad + \frac{1}{2}z_{in}^2 + \frac{1}{2}\bar{\varepsilon}_{in}^2, \end{aligned} \tag{80}$$

where $\theta_{in} = \|W_{in}\|^2$.

Substituting (80) into (78) yields that

$$\begin{aligned} \dot{V}_{in} \le& z_{in}Y_{in}\chi_i(u_i)u_i(t) \\ &+ \frac{1}{2\mu_{in}}z_{in}^2\theta_{in}\varphi_{in}^T(Z_{in})\varphi_{in}(Z_{in}) \\ &+ \frac{1}{2}\mu_{in} - \frac{1}{2}z_{in}^2 + \frac{1}{2}\bar{\varepsilon}_{in}^2 + \frac{1}{2}\bar{d}_{in}^2 \\ &- \frac{1}{\eta_{in}}\tilde{\theta}_{in}\dot{\hat{\theta}}_{in} - l_i\tilde{\sigma}_{in}\dot{\hat{\sigma}}_{in}. \end{aligned} \tag{81}$$

Choose the saturation control input u_i and the adaptive rules $\dot{\hat{\theta}}_{in}$, $\dot{\hat{\sigma}}_{in}$ as

$$u_i = \hat{\sigma}_{in}\hat{\alpha}_{in}, \tag{82}$$

$$\hat{\alpha}_{in} = -k_{in}z_{in}^{2\beta_1-1} - \frac{1}{2\mu_{in}}z_{in}\hat{\theta}_{in}\varphi_{in}^T(Z_{in})\varphi_{in}(Z_{in}), \tag{83}$$

$$\dot{\hat{\theta}}_{in} = -\gamma_{in}^{(1)}\hat{\theta}_{in}^{2\beta_1-1} - \gamma_{in}^{(2)}\hat{\theta}_{in}^{2\beta_2-1} + \frac{\eta_{in}}{2\mu_{in}}z_{in}^2\varphi_{in}^T(Z_{in})\varphi_{in}(Z_{in}), \tag{84}$$

and

$$\dot{\hat{\sigma}}_{in} = -m_{in}^{(1)}\hat{\sigma}_{in}^{2\beta_1-1} - m_{in}^{(2)}\hat{\sigma}_{in}^{2\beta_2-1} - z_{in}\hat{\alpha}_{in}, \tag{85}$$

where $k_{in}, \gamma_{in}^{(1)}, \gamma_{in}^{(2)}, m_{in}^{(1)}, m_{in}^{(2)} > 0$, $\beta_1 > 1$, and $0 < \beta_2 < 1$ are design parameters.

Noting that $z_{in}\hat{\alpha}_{in} \leq 0$ and $\hat{\sigma}_{in} \geq 0$, one has

$$\begin{aligned} z_{in}Y_{in}\chi_i(u_i)u_i(t) &= z_{in}Y_{in}\chi_i(u_i)\hat{\sigma}_{in}\hat{\alpha}_{in} \\ &\leq z_{in}\underline{Y}_{in}o_i\hat{\sigma}_{in}\hat{\alpha}_{in} \\ &= z_{in}l_i\left(\frac{1}{l_i} - \tilde{\sigma}_{in}\right)\hat{\alpha}_{in} \\ &= z_{in}\hat{\alpha}_{in} - z_{in}l_i\tilde{\sigma}_{in}\hat{\alpha}_{in}. \end{aligned} \tag{86}$$

From (82)–(86), one gets that

$$\begin{aligned} \dot{V}_{in} \leq& -k_{in}z_{in}^{2\beta_1} - \frac{1}{2}z_{in}^2 + \frac{\gamma_{in}^{(1)}}{\mu_{in}}\tilde{\theta}_{in}\hat{\theta}_{in}^{2\beta_1-1} \\ &+ \frac{\gamma_{in}^{(2)}}{\mu_{in}}\tilde{\theta}_{in}\hat{\theta}_{in}^{2\beta_2-1} + m_{in}^{(1)}l_i\tilde{\sigma}_{in}\hat{\sigma}_{in}^{2\beta_1-1} \\ &+ m_{in}^{(2)}l_i\tilde{\sigma}_{in}\hat{\sigma}_{in}^{2\beta_2-1} + \omega_{in} \\ =& -k_{in}z_{in}^{2\beta_1} - \frac{1}{2}z_{in}^2 + \frac{\gamma_{in}^{(1)}}{\mu_{in}}\tilde{\theta}_{in}\hat{\theta}_{in}^{2\beta_1-1} \\ &+ \frac{\gamma_{in}^{(2)}}{\mu_{in}}\tilde{\theta}_{in}\hat{\theta}_{in}^{2\beta_2-1} + \frac{m_{in}^{(1)}}{\sigma_{in}}\tilde{\sigma}_{in}\hat{\sigma}_{in}^{2\beta_1-1} \\ &+ \frac{m_{in}^{(2)}}{\sigma_{in}}\tilde{\sigma}_{in}\hat{\sigma}_{in}^{2\beta_2-1} + \omega_{in}, \end{aligned} \tag{87}$$

where $\omega_{in} = \frac{1}{2}\mu_{in} + \frac{1}{2}\bar{\varepsilon}_{in}^2 + \frac{1}{2}\bar{d}_{in}^2$.

4. Stability Analysis

In this section, we give the main results summarized in Theorem 1 and conduct the fixed-time stability analysis of the closed-loop system.

Choose the overall Lyapunov function as

$$V = \sum_{i=1}^{N}\left(\sum_{q=1}^{n}V_{iq}\right). \tag{88}$$

From (41), (57), (73), and (87), one gets

$$\begin{aligned} \dot{V} \leq \sum_{i=1}^{N}\sum_{q=1}^{n}\Bigg(& -k_{iq}z_{iq}^{2\beta_1} + \omega_{iq} + \frac{\gamma_{iq}^{(1)}}{\mu_{iq}}\tilde{\theta}_{iq}\hat{\theta}_{iq}^{2\beta_1-1} \\ &+ \frac{\gamma_{iq}^{(2)}}{\mu_{iq}}\tilde{\theta}_{iq}\hat{\theta}_{iq}^{2\beta_2-1} + \frac{m_{iq}^{(1)}}{\sigma_{iq}}\tilde{\sigma}_{iq}\hat{\sigma}_{iq}^{2\beta_1-1} \\ &+ \frac{m_{iq}^{(2)}}{\sigma_{iq}}\tilde{\sigma}_{iq}\hat{\sigma}_{iq}^{2\beta_2-1}\Bigg). \end{aligned} \tag{89}$$

By using Lemmas 2.7 and 2.8, one has

$$-\frac{1}{2}\sum_{q=1}^{n}k_{iq}z_{iq}^{2\beta_1} \leq -\frac{k_i}{n^{\beta_1-1}}\left(\sum_{q=1}^{n}z_{iq}^2\right)^{\beta_1} \tag{90}$$

and

$$-\left(\sum_{q=1}^{n}z_{iq}^2\right)^{\beta_1} \leq -\left(\sum_{q=1}^{n}z_{iq}^2\right)^{\beta_2} + \frac{\beta_1-\beta_2}{\beta_1}\left(\frac{\beta_2}{\beta_1}\right)^{\frac{\beta_2}{\beta_1-\beta_2}}, \tag{91}$$

where $k_i = \min_{1\leq q\leq n}k_{iq}$, $0 < \beta_2 < 1$.

Form (90) and (91), one has

$$\begin{aligned} -\sum_{q=1}^{n}k_{iq}z_{iq}^{2\beta_1} \leq& -\frac{k_i}{n^{\beta_1-1}}\left(\sum_{q=1}^{n}z_{iq}^2\right)^{\beta_1} \\ &- \frac{k_i}{n^{\beta_1-1}}\left(\sum_{q=1}^{n}z_{iq}^2\right)^{\beta_2} \\ &+ \frac{k_i}{n^{\beta_1-1}}\frac{\beta_1-\beta_2}{\beta_1}\left(\frac{\beta_2}{\beta_1}\right)^{\frac{\beta_2}{\beta_1-\beta_2}}. \end{aligned} \tag{92}$$

From Lemmas 2.10 and 2.11, we have

$$\tilde{\theta}_{iq}\hat{\theta}_{iq}^{2\beta_2-1} \leq m_1\theta_{iq}^{2\beta_2} - m_2\tilde{\theta}_{iq}^{2\beta_2}, \tag{93}$$

and

$$\tilde{\theta}_{iq}\hat{\theta}_{iq}^{2\beta_1-1} = \tilde{\theta}_{iq}(\theta_{iq} - \tilde{\theta}_{iq})^{2\beta_1-1}$$

$$\leq \theta_{iq}^{2\beta_1} - \tilde{\theta}_{iq}^{2\beta_1}, \tag{94}$$

where $m_1 = \frac{1}{2\beta_2}[1 - 2^{2\beta_2-2} + \frac{2\beta_2-1}{2\beta_2} + \frac{2^{2\beta_2-1}[1-(2\beta_2-1)^2]}{2\beta_2}]$, $m_2 = \frac{2^{2\beta_2-2}}{2\beta_2}[1 - 2^{(2\beta_2-1)(2\beta_2-2)}]$.

Similarly,

$$\tilde{\sigma}_{iq}\hat{\sigma}_{iq}^{2\beta_2-1} \leq m_1\sigma_{iq}^{2\beta_2} - m_2\tilde{\sigma}_{iq}^{2\beta_2}, \tag{95}$$

and

$$\begin{aligned}\tilde{\sigma}_{iq}\hat{\sigma}_{iq}^{2\beta_1-1} &= \tilde{\sigma}_{iq}(\sigma_{iq} - \tilde{\sigma}_{iq})^{2\beta_1-1} \\ &\leq \sigma_{iq}^{2\beta_1} - \tilde{\sigma}_{iq}^{2\beta_1}.\end{aligned} \tag{96}$$

Substituting (90)–(96) into (89), one obtains that

$$\begin{aligned}\dot{V} \leq \sum_{i=1}^{N}\Bigg[&-\frac{k_i}{n^{\beta_1-1}}\left(\sum_{q=1}^{n} z_{iq}^2\right)^{\beta_1} \\ &- \frac{k_i}{n^{\beta_1-1}}\left(\sum_{q=1}^{n} z_{iq}^2\right)^{\beta_2} + \frac{k_i}{n^{\beta_1-1}}\frac{\beta_1-\beta_2}{\beta_1} \\ &\times\left(\frac{\beta_2}{\beta_1}\right)^{\frac{\beta_2}{\beta_1-\beta_2}} + \sum_{q=1}^{n}\frac{\gamma_{iq}^{(1)}}{\mu_{iq}}\theta_{iq}^{2\beta_1} \\ &- \sum_{q=1}^{n}\frac{\gamma_{iq}^{(1)}}{\mu_{iq}}\tilde{\theta}_{iq}^{2\beta_1} + \sum_{q=1}^{n}\frac{\gamma_{iq}^{(2)}}{\mu_{iq}}m_1\theta_{iq}^{2\beta_2} \\ &- \sum_{q=1}^{n}\frac{\gamma_{iq}^{(2)}}{\mu_{iq}}m_2\tilde{\theta}_{iq}^{2\beta_2} + \sum_{q=1}^{n}\frac{m_{iq}^{(1)}}{\sigma_{iq}}\sigma_{iq}^{2\beta_1} \\ &- \sum_{q=1}^{n}\frac{m_{iq}^{(1)}}{\sigma_{iq}}\tilde{\sigma}_{iq}^{2\beta_1} + \sum_{q=1}^{n}\frac{m_{iq}^{(2)}}{\sigma_{iq}}m_1\sigma_{iq}^{2\beta_2} \\ &- \sum_{q=1}^{n}\frac{m_{iq}^{(2)}}{\sigma_{iq}}m_2\tilde{\sigma}_{iq}^{2\beta_2} + \sum_{q=1}^{n}w_{iq}\Bigg] \\ = \sum_{i=1}^{N}\Bigg[&-\frac{2^{\beta_1}k_i}{n^{\beta_1-1}}\left(\frac{1}{2}\sum_{q=1}^{n} z_{iq}^2\right)^{\beta_1} \\ &- \sum_{q=1}^{n}2^{\beta_1}\mu_{iq}^{\beta_1-1}\gamma_{iq}^{(1)}\left(\frac{\tilde{\theta}_{iq}^2}{2\mu_{iq}}\right)^{\beta_1} \\ &- \sum_{q=1}^{n}2^{\beta_1}\mu_{iq}^{\beta_1-1}m_{iq}^{(1)}\left(\frac{\tilde{\sigma}_{iq}^2}{2\sigma_{iq}}\right)^{\beta_1} \\ &- \frac{2^{\beta_2}k_i}{n^{\beta_1-1}}\left(\frac{1}{2}\sum_{q=1}^{n} z_{iq}^2\right)^{\beta_2} \\ &- \sum_{q=1}^{n}2^{\beta_2}\mu_{iq}^{\beta_2-1}\gamma_{iq}^{(2)}m_2\left(\frac{\tilde{\theta}_{iq}^2}{2\mu_{iq}}\right)^{\beta_2} \\ &- \sum_{q=1}^{n}2^{\beta_2}\mu_{iq}^{\beta_2-1}m_{iq}^{(2)}m_2\left(\frac{\tilde{\sigma}_{iq}^2}{2\sigma_{iq}}\right)^{\beta_2} \\ &+ \frac{k_i}{n^{\beta_1-1}}\frac{\beta_1-\beta_2}{\beta_1}\left(\frac{\beta_2}{\beta_1}\right)^{\frac{\beta_2}{\beta_1-\beta_2}} + \sum_{q=1}^{n}\frac{\gamma_{iq}^{(1)}}{\mu_{iq}}\theta_{iq}^{2\beta_1} \\ &+ \sum_{q=1}^{n}\frac{\gamma_{iq}^{(2)}}{\mu_{iq}}\theta_{iq}^{2\beta_2} + \sum_{q=1}^{n}\frac{m_{iq}^{(1)}}{\sigma_{iq}}\sigma_{iq}^{2\beta_1} \\ &+ \sum_{q=1}^{n}\frac{m_{iq}^{(2)}}{\sigma_{iq}}\sigma_{iq}^{2\beta_2} + \sum_{q=1}^{n}w_{iq}\Bigg].\end{aligned} \tag{97}$$

It follows from (97) that

$$\begin{aligned}\dot{V} \leq \sum_{i=1}^{N}\Bigg\{&-\rho_{i1}\left[\left(\frac{1}{2}\sum_{q=1}^{n} z_{iq}^2\right)^{\beta_1} + \left(\sum_{q=1}^{n}\frac{\tilde{\theta}_{iq}^2}{2\mu_{iq}}\right)^{\beta_1}\right. \\ &\left.+\left(\sum_{q=1}^{n}\frac{\tilde{\sigma}_{iq}^2}{2\sigma_{iq}}\right)^{\beta_1}\right] - \rho_{i2}\left[\left(\frac{1}{2}\sum_{q=1}^{n} z_{iq}^2\right)^{\beta_2}\right. \\ &\left.+\left(\sum_{q=1}^{n}\frac{\tilde{\theta}_{iq}^2}{2\mu_{iq}}\right)^{\beta_2} + \left(\sum_{q=1}^{n}\frac{\tilde{\sigma}_{iq}^2}{2\sigma_{iq}}\right)^{\beta_2}\right] + M_i\Bigg\},\end{aligned} \tag{98}$$

where

$$\rho_{i1} = \min_{1\leq q\leq n}\left\{\frac{k_i 2^{\beta_1}}{n^{\beta_1-1}}, 2^{\beta_1}\mu_{iq}^{\beta_1-1}\gamma_{iq}^{(1)}, 2^{\beta_1}\mu_{iq}^{\beta_1-1}m_{iq}^{(1)}\right\},$$

$$\rho_{i2} = \min_{1\leq q\leq n}\left\{\frac{k_i 2^{\beta_2}}{n^{\beta_1-1}}, 2^{\beta_2}\mu_{iq}^{\beta_2-1}\gamma_{iq}^{(2)}, 2^{\beta_2}\mu_{iq}^{\beta_2-1}m_{iq}^{(2)}\right\},$$

and

$$\begin{aligned}M_i = &\frac{k_i}{n^{\beta_1-1}}\frac{\beta_1-\beta_2}{\beta_1}\left(\frac{\beta_2}{\beta_1}\right)^{\frac{\beta_2}{\beta_1-\beta_2}} + \sum_{q=1}^{n}\frac{\gamma_{iq}^{(1)}}{\mu_{iq}}\theta_{iq}^{2\beta_1} \\ &+ \sum_{q=1}^{n}\frac{\gamma_{iq}^{(2)}}{\mu_{iq}}\theta_{iq}^{2\beta_2} + \sum_{q=1}^{n}\frac{m_{iq}^{(1)}}{\sigma_{iq}}\sigma_{iq}^{2\beta_1}\end{aligned}$$

$$+\sum_{q=1}^{n}\frac{m_{iq}^{(2)}}{\sigma_{iq}}\sigma_{iq}^{2\beta_2}+\sum_{q=1}^{n}w_{iq}. \tag{99}$$

Let $\rho^{(1)} = \min_{1\le q\le n}\rho_{i1}$, $\rho^{(2)} = \min_{1\le q\le n}\rho_{i2}$, by using Lemma 2.8, one has

$$\begin{aligned}\dot{V} &\le \sum_{i=1}^{N}\left\{-\frac{\rho_{i1}}{3^{\beta_1-1}}\left[\sum_{q=1}^{n}\left(\frac{1}{2}z_{iq}^2+\frac{\tilde{\theta}_{iq}^2}{2\mu_{iq}}+\frac{\tilde{\sigma}_{iq}^2}{2\sigma_{iq}}\right)\right]^{\beta_1}\right.\\ &\quad\left.-\rho_{i2}\left[\sum_{q=1}^{n}\left(\frac{1}{2}z_{iq}^2+\frac{\tilde{\theta}_{iq}^2}{2\mu_{iq}}+\frac{\tilde{\sigma}_{iq}^2}{2\sigma_{iq}}\right)\right]^{\beta_2}+M_i\right\}\\ &\le \sum_{i=1}^{N}\left\{-\frac{\rho^{(1)}}{3^{\beta_1-1}}\left[\sum_{q=1}^{n}\left(\frac{1}{2}z_{iq}^2+\frac{\tilde{\theta}_{iq}^2}{2\mu_{iq}}+\frac{\tilde{\sigma}_{iq}^2}{2\sigma_{iq}}\right)\right]^{\beta_1}\right.\\ &\quad\left.-\rho^{(2)}\left[\sum_{q=1}^{n}\left(\frac{1}{2}z_{iq}^2+\frac{\tilde{\theta}_{iq}^2}{2\mu_{iq}}+\frac{\tilde{\sigma}_{iq}^2}{2\sigma_{iq}}\right)\right]^{\beta_2}+M_i\right\}\\ &\le -\frac{\rho^{(1)}}{(3N)^{\beta_1-1}}\left[\sum_{i=1}^{N}\sum_{q=1}^{n}\left(\frac{1}{2}z_{iq}^2+\frac{\tilde{\theta}_{iq}^2}{2\mu_{iq}}+\frac{\tilde{\sigma}_{iq}^2}{2\sigma_{iq}}\right)\right]^{\beta_1}\\ &\quad-\rho^{(2)}\left[\sum_{i=1}^{N}\sum_{q=1}^{n}\left(\frac{1}{2}z_{iq}^2+\frac{\tilde{\theta}_{iq}^2}{2\mu_{iq}}+\frac{\tilde{\sigma}_{iq}^2}{2\sigma_{iq}}\right)\right]^{\beta_2}+M,\end{aligned} \tag{100}$$

where $M=\sum_{i=1}^{N}M_i$.

Therefore, one has

$$\dot{V}\le -\rho_1V^{\beta_1}-\rho_2V^{\beta_2}+M, \tag{101}$$

where $\rho_1=\frac{\rho^{(1)}}{(3N)^{\beta_1-1}}$, $\rho_2=\rho^{(2)}$.

Theorem 4.1. *Consider system* (1) *with senor hysteresis* (5), *and input saturations* (2). *Under Assumptions* 2.1–2.3, *applying the adaptive laws* (38), (39), (54), (55), (70), (71), (84), *and* (85), *and the virtual controllers* (36), (52), (68), *and* (82), *the practical fixed-time stability of the closed-loop system is ensured. Moreover, the residual set is obtained as*

$$\Omega=\left\{x=(\hat{x}_{iq},\tilde{\theta}_{iq},\tilde{\sigma}_{iq})^T \mid V\le \min\left(\left[\frac{M}{\rho_1(1-\theta)}\right]^{\frac{1}{\beta_1}},\left[\frac{M}{\rho_2(1-\theta)}\right]^{\frac{1}{\beta_2}}\right)\triangleq\bar{V}\right\}, \tag{102}$$

and the settling time T satisfies

$$T\le T_{\max}=\frac{1}{\rho_1\theta(\beta_1-1)}+\frac{1}{\rho_2\theta(1-\beta_2)}, \tag{103}$$

where $\theta\in(0,1)$ is a positive constant, and the following statements are ensured:

(i) *Error signals z_{iq}, adaptive parameter errors $\tilde{\theta}_{iq}$, and $\tilde{\sigma}_{iq}$ satisfy that*

$$|z_{iq}|\le\sqrt{2\bar{V}}, \tag{104}$$

$$|\tilde{\theta}_{iq}|\le\sqrt{2\mu_{iq}\bar{V}}, \tag{105}$$

and

$$|\tilde{\sigma}_{iq}|\le\sqrt{\frac{2\bar{V}}{\underline{Y}_{iq}}}, \tag{106}$$

where $\bar{V}>0$ is a constant.

(ii) *The tracking errors $y_i-y_0, i = 1,\dots,N$, converge to a bounded neighborhood around the origin in a fixed time.*

Proof. It follows from (101) and Lemma 2.6 that the closed-loop system is practically fixed-time stable, and (102) and (103) hold.

In the following, we prove that (i) and (ii) hold.

(i) From (88) and (102), we have

$$\frac{1}{2}z_{iq}^2\le V\le\bar{V},\quad q=1,\dots,n. \tag{107}$$

Thus,

$$|z_{iq}|\le\sqrt{2\bar{V}}. \tag{108}$$

Similarly,

$$|\tilde{\theta}_{iq}|\le\sqrt{2\mu_{iq}\bar{V}} \tag{109}$$

and

$$|\tilde{\sigma}_{iq}|\le\sqrt{\frac{2\bar{V}}{\underline{Y}_{iq}}}. \tag{110}$$

(ii) Form (20), we have

$$\|\hat{y}\|-\|1_Ny_0\|\le\|\hat{y}-1_Ny_0\|\le\frac{\|z_1\|}{\underline{\sigma}(\mathcal{L}+\mathcal{B})}. \tag{111}$$

Therefore,

$$\|\hat{y}\|\le\frac{\|z_1\|}{\underline{\sigma}(\mathcal{L}+\mathcal{B})}+\sqrt{N}|y_0|. \tag{112}$$

Let $y = [y_1,y_2,\dots,y_N]^T = [x_{11},x_{21},\dots,x_{N1}]^T$, $\zeta = \mathrm{diag}\{\zeta_{11},\zeta_{21},\dots,\zeta_{N1}\}$, $\Lambda=\mathrm{diag}\{\lambda_{11},\dots,\lambda_{N1}\}$, $R=[r_{11},\dots,r_{N1}]^T$.

It follows from (5) that

$$\hat{y}=\zeta y+\Lambda R. \tag{113}$$

Thus,

$$y=\zeta^{-1}(\hat{y}-\Lambda R), \tag{114}$$

$$\begin{aligned}\|y-1_Ny_0\|&=\|\zeta^{-1}(\hat{y}-\Lambda R)-1_Ny_0\|\\ &\le\|\zeta^{-1}\hat{y}\|+\|\zeta^{-1}\Lambda R\|+\|1_Ny_0\|\end{aligned}$$

$$\leq \frac{1}{\min \zeta_{i1}}\left(\frac{\|z_1\|}{\underline{\sigma}(\mathcal{L}+\mathcal{B})}+\sqrt{N}|y_0|\right) + \sqrt{N}\max_i\left\{\frac{\lambda_{i1}\bar{r}_{i1}}{\zeta_{i1}}\right\}+\sqrt{N}|y_0| \leq c_1\sqrt{2N\bar{V}}+c_2|y_0|+c_3, \quad (115)$$

where $c_1 = \frac{1}{\min \zeta_{i1}\underline{\sigma}(\mathcal{L}+\mathcal{B})}$, $c_2 = (\frac{1}{\min \zeta_{i1}}+1)\sqrt{N}$, and $c_3 = \sqrt{N}\max_i\{\frac{\lambda_{i1}\bar{r}_{i1}}{\zeta_{i1}}\}$ are the constants.

Consequently, the following agents can track the leader agent in a fixed time with bounded errors. This completes the proof. □

Remark 4.2. In MASs considered in this paper, each agent is an n-order system. As the order of each agents increases, the time and space complexity of the proposed algorithm will increase accordingly. By selecting the adaptive parameter as the two-norm of the unknown parameter vector of the RBFNN, the dimension of the adaptive parameter vector is greatly reduced. When the algorithm is applied to each agent, which is actually a $3n$-order ordinary differential adaptive system, the time and space complexity of the algorithm is significantly decreased. Furthermore, the proposed algorithm is distributed, and each agent can carry out distributed computing based on network communication. Taking the second-order MASs that have been widely studied and applied in practice as an example, each agent is actually a sixth-order adaptive system. With the rapid development of current computer software and hardware technologies, the algorithm proposed in this paper is fully capable of being applied to practical systems for real-time computing and ensuring computational efficiency.

Remark 4.3. It can be seen from (101) that larger parameters ρ_1, ρ_2 and a smaller parameters M will result in a faster convergence rate and a smaller error bound, respectively. Although choosing larger parameters $k_{i1}, \mu_{i1}, \gamma_{i1}^{(1)}, \gamma_{i1}^{(2)}, m_{i1}^{(1)}$, and $m_{i1}^{(2)}$ will result in a faster convergence rate, it may lead to an increase in the error bound. Therefore, we need to find a balance between a faster convergence rate and a smaller error bound.

5. Simulation

In this section, we provide an example to confirm the validity of our theoretical results.

Consider an MAS including a leader and four one-link manipulators.

The four manipulators are described by

$$\tau_i^{(1)}\ddot{p}_i+\tau_i^{(2)}\dot{p}_i+\tau_i^{(3)}\sin(p_i) = \mathrm{sat}(u_i)+\varpi_i, \quad i=1,2,3,4, \quad (116)$$

where $\tau_i^{(1)}, \tau_i^{(2)}, \tau_i^{(3)}$ are unknown constants. $\ddot{p}_i$, $\dot{p}_i$, and p_i are the link acceleration, the angular velocity and the angle of the rigid link, respectively. ϖ_i is the torque disturbance.

Letting $x_{i1}=p_i$ and $x_{i2}=\dot{p}_i$, form (116), one has

$$\begin{cases}\dot{x}_{i1}=x_{i2},\\ \dot{x}_{i2}=\dfrac{1}{\tau_i^{(1)}}\mathrm{sat}(u_i(t))-\dfrac{\tau_i^{(2)}}{\tau_i^{(1)}}x_{i2} \\ \qquad -\dfrac{\tau_i^{(3)}}{\tau_i^{(1)}}\sin(x_{i1})+\dfrac{1}{\tau_i^{(1)}}\varpi_i,\\ y_i=x_{i1}, \quad i=1,2,3,4.\end{cases}$$

Here, as an example, let $\tau_i^{(1)}=\tau_i^{(2)}=1$, $\tau_i^{(3)}=2$, and $\varpi_i=0.4\cos(t)$. The interactions among agents are shown in Fig. 1. Set the initial values as $x_{11}=x_{31}=0.15$, $x_{12}=x_{22}=x_{32}=x_{42}=0.1$, $x_{21}=x_{41}=0.2$, $\hat{\theta}_{11}=0.3$, $\hat{\theta}_{21}=0.25$, $\hat{\theta}_{31}=0.4$, $\hat{\theta}_{41}=0.5$, $\hat{\theta}_{i2}=0.3$, $r_{i1}=r_{i2}=0.1$, $\hat{\sigma}_{11}=0$, $\hat{\sigma}_{21}=2.5$, $\hat{\sigma}_{31}=0.3$, $\hat{\sigma}_{41}=3$, and $\hat{\sigma}_{i2}=0.5$. The leader signal is given as $y_0=\sin(t)$. Select the main parameters as $\beta_1=\frac{101}{99}$, $\beta_2=\frac{79}{81}$, $\mu_{i1}=\mu_{i2}=5$, $\eta_{i1}=\eta_{i2}=5$, $m_{11}^{(1)}=m_{21}^{(1)}=m_{31}^{(1)}=0.03$, $m_{41}^{(1)}=0.04$, $m_{i2}^{(1)}=m_{i1}^{(2)}=m_{i2}^{(2)}=0.1$, $\gamma_{i1}^{(1)}=0.03$, $\gamma_{i2}^{(1)}=0.2$, $\gamma_{i1}^{(2)}=0.01$, $\gamma_{i2}^{(2)}=0.5$, $k_{31}=12$, $k_{11}=k_{21}=k_{41}=10$, $k_{22}=k_{32}=k_{42}=30$, $k_{12}=20$, $u_{1m}=u_{3m}=2.5$, $u_{2m}=5$, $u_{4m}=4$, $u_{1M}=u_{3M}=u_{4M}=3$, $u_{2M}=10$, $h_{i,11}=h_{i,21}=1$, $h_{i,12}=h_{i,22}=2$, $h_{i,13}=h_{i,23}=0.5$, $\zeta_{11}=\zeta_{31}=\zeta_{12}=\zeta_{32}=0.1$, $\zeta_{21}=\zeta_{41}=0.12$, $\zeta_{22}=\zeta_{42}=0.18$, and $\lambda_{i1}=\lambda_{i2}=0.9$, $i=1,2,3,4$.

For each unknown function, the RBFNN contains five nodes. The centers of the Gaussian function are uniformly selected within the interval $[-2,2]$, and its widths are chosen to be 2.

Within 40 s of simulation time, Figs. 2–9 show the simulation results. Figure 2 exhibits the curves of y_i and y_0, and the corresponding error curves are shown in Fig. 3. It can be seen from Figs. 2 and 3 that the following agents can track the leader agent with acceptable errors. The curves of the adaptive parameters $\hat{\theta}_{i1}$ and $\hat{\sigma}_{i1}$ are shown in Figs. 4 and 5. Figures 6–9 show the curves of u_i and $\mathrm{sat}_i(u_i)$. From the expanded details of Figs. 6–9, we can see that when a larger control input (blue line) is required, the actual saturation control input (red line) can still provide the expected control effect.

Remark 5.1. The challenges and limitations that the proposed adaptive fixed-time scheme may encounter in practical applications are mainly manifested in two aspects. On

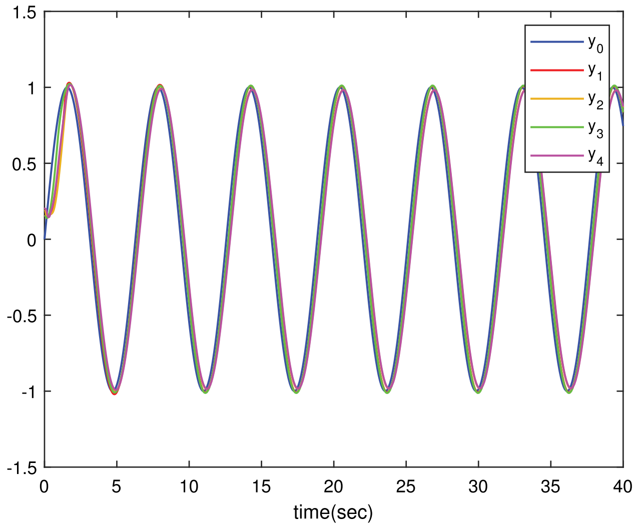

Fig. 2. The trajectories of y_i and y_0.

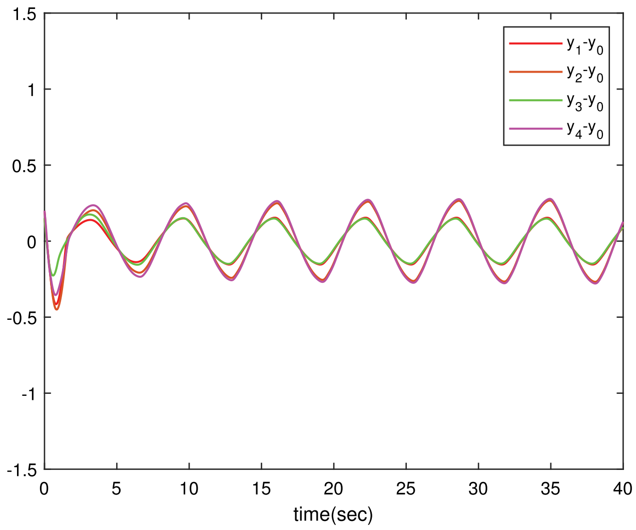

Fig. 3. Consensus tracking errors e_{i1}.

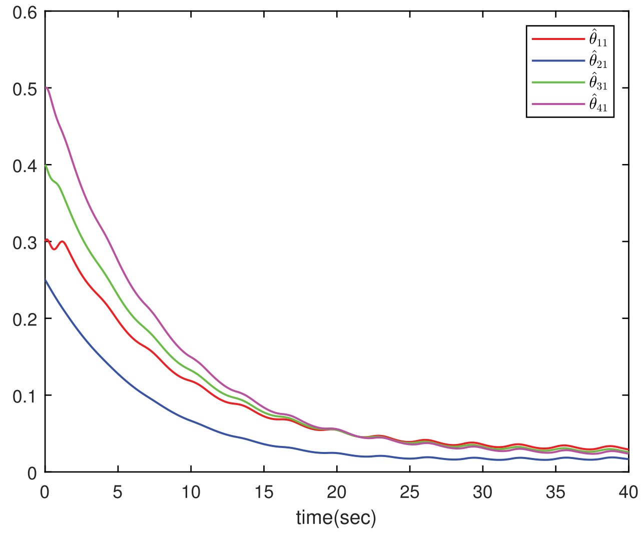

Fig. 4. Adaptive parameters $\hat{\theta}_{11}, \hat{\theta}_{21}, \hat{\theta}_{31}, \hat{\theta}_{41}$.

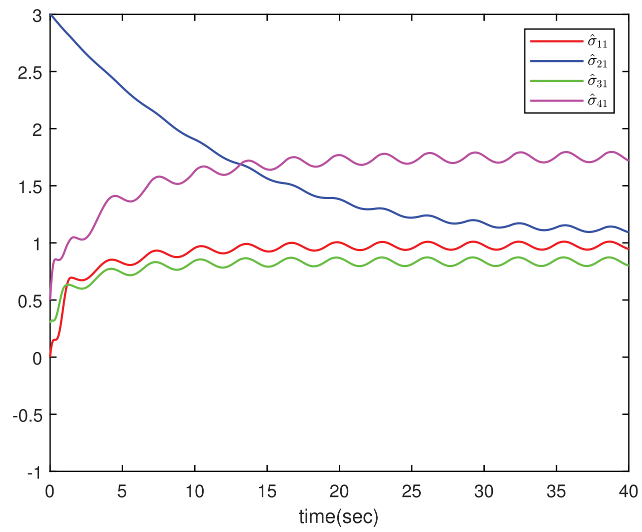

Fig. 5. Adaptive parameters $\hat{\sigma}_{11}, \hat{\sigma}_{21}, \hat{\sigma}_{31}, \hat{\sigma}_{41}$.

the one hand, the construction of the RBFNN requires prior understanding of the system, for example, the approximate range of system states and the selection of the centers and widths of the activation functions of the RBFNN. These are all influenced by expert knowledge and affect the approximation accuracy of the RBFNN. On the other hand, the communication between agents may be affected by some uncertain factors, such as communication time-delays and network attacks. These uncertain factors not considered in this paper may lead to the failure of the algorithm and can be further explored in our future research work.

Remark 5.2. According to (102), the upper bound of the settling time can be calculated to be $T_{\max} = 300$ s. The estimation of the upper bound of the settling time is evidently somewhat conservative. Judging from the simulation results, the system can achieve practical fixed-time stability within approximately 40 s, that is, converge to a bounded neighborhood of the origin. How to obtain a more compact upper bound of the settling time can be a research topic in the future.

Remark 5.3. By conducting simulations, comparisons have been made with several other types of stabilities, such as practical asymptotic stability and practical finite-time stability. By observing the convergence curves, it is found that there is no significant difference in their convergence times. According to [28], Lemma 3], if a system is practically asymptotically stable, then it is also practically finite-time stable. However, compared with practically asymptotically stable

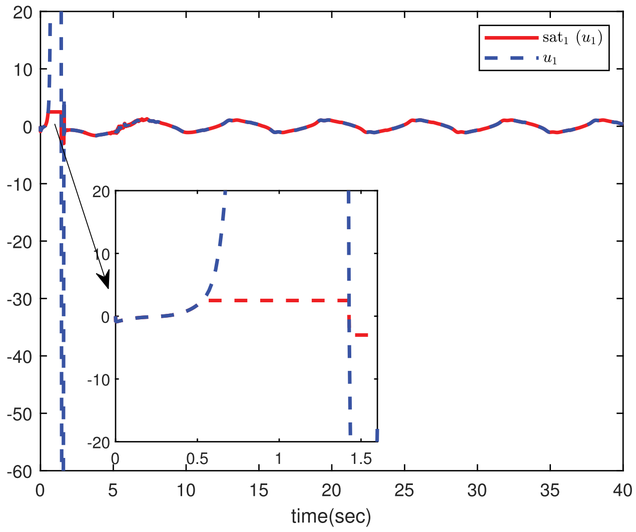

Fig. 6. Saturated controller $\mathrm{sat}_1(u_1)$ and its input u_1.

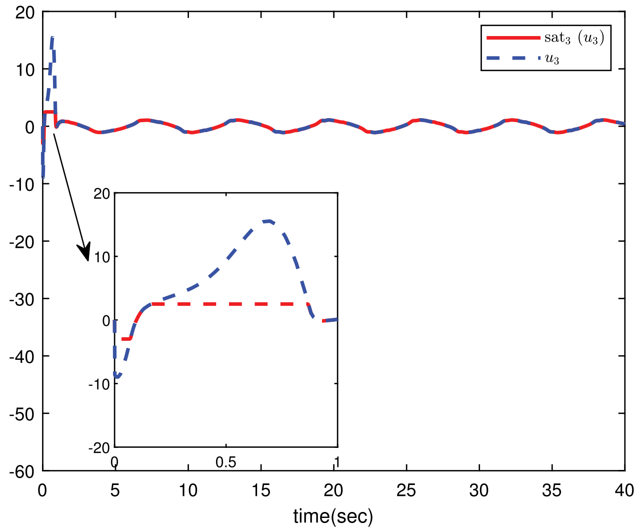

Fig. 8. Saturated controller $\mathrm{sat}_3(u_3)$ and its input u_3.

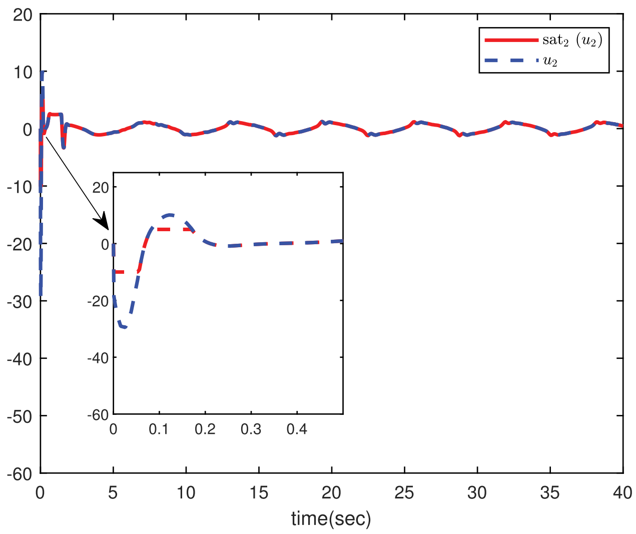

Fig. 7. Saturated controller $\mathrm{sat}_2(u_2)$ and its input u_2.

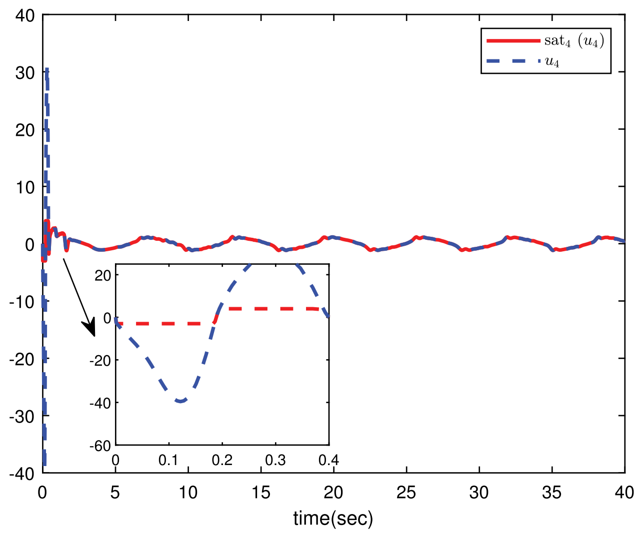

Fig. 9. Saturated controller $\mathrm{sat}_4(u_4)$ and its input u_4.

systems, in practically finite-time stable systems, the estimation of the bound of the convergence region is more conservative [28]. Compared with the system with practical finite-time stability, the advantage of the practical fixed-time control algorithm lies in the fact that the upper bound of the settling time is independent of the initial state of the system.

Remark 5.4. As mentioned in Remark 4.2, the algorithm proposed in this paper is distributed, and each agent is capable of performing distributed computations based on local information. Even if the number of follower agents increases, it will not exert a significant impact on the computational complexity. For example, in the case where the communication network assumes a chain structure as shown in Fig. 10, regardless of the scale of the communication network, at any moment, each agent only needs to communicate with one neighboring agent.

Fig. 10. Chain structure network.

6. Conclusion

This work discusses the fixed-time control issue of MASs with unknown nonlinear dynamics and external disturbances subject to input saturation and unknown sensor hysteresis. Using the backstepping design approach, an adaptive fixed-time control scheme is presented for the considered system to achieve consensus tracking. The practical fixed-time stability of the closed-loop system is guaranteed by stability analysis. The simulation results also show that the designed control scheme is effective. In future research, the related topics can be further investigated in systems suffering network attacks, etc.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT Authorship Contribution Statement

Zifei Zhou: Conceptualization, Methodology, Investigation, Writing–original draft. Hui Yu: Validation, Supervision, Writing–review and editing.

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