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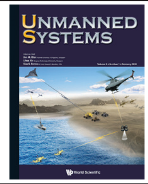

An Overview of Recent Advances in Distributed Coordination of Multi-Agent Systems

Ruohan Yang[*,‡], Lu Liu[†,§], Gang Feng[†,¶]

[*]*School of Electronics and Information,*
Northwestern Polytechnical University,
Xian 710072, P. R. China

[†]*Department of Biomedical Engineering,*
City University of Hong Kong, Hong Kong

Distributed coordination of multi-agent systems (MASs) has received considerable attention and advanced rapidly in the past years. This paper reviews the recent results on cooperative control of MASs, and in particular cooperative control under various agent dynamics, under communication transmission delays, and under event-triggered strategies. In addition, different coordination tasks, such as consensus, cooperative output regulation, formation control, and containment control, are also taken into consideration. Finally, some open problems on cooperative control of MASs are suggested.

Keywords: Distributed coordination; multi-agent systems; consensus; cooperative output regulation; communication delays; event-triggered control.

1. Introduction

MASs have been used to describe some collective behaviors of various social animals, such as swarming of bees, schooling of fish, and flocking of birds [1–3], and also some man-made systems, such as networked mobile robots, sensor networks [4–6]. MASs refer to systems composed of a number of individual dynamic subsystems named agents, interconnected through some kind of networks. Compared with single systems, MASs have higher efficiency, greater capabilities and lower cost. Moreover, MASs enjoy appealing properties of robustness and scalability, which can tolerate failures of agents and allow addition of new agents. Due to these advantages, MASs have been widely adopted in many areas including multiple spacecraft vehicles [7], biological systems [8], mobile robots [5], sensor networks [6], flocking [9,10] and so on.

In the past decades, various cooperative control problems of MASs have been extensively investigated. These problems include consensus [11–14], synchronization [15,16], cooperative output regulation [17–19], formation [20–22], containment control [23,24] and so on. There are typically two commonly used approaches to cooperative control: one is the centralized approach, and the other is the distributed approach. The centralized control approach assumes that a central station is sufficiently powerful to control a whole group of agents. In contrast to the centralized control approach, the distributed control approach does not require a central station, at the cost of complexity in structure and organization, and thus provides some distinctive advantages such as low operational cost and flexible scalability. A key to the distributed control approach is to design distributed controllers for individual agents such that all the agents can achieve a common goal of interest while each agent has only access to the information of its neighboring agents.

In cooperative control of real-world MASs, communication time delays often occur when information is transmitted among agents due to several reasons, for example limited information transmission speed, extra time for information

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Email Addresses: [‡]*ruohanyang@nwpu.edu.cn*, [§]*luliu45@cityu.edu.hk*, [¶]*megfeng@cityu.edu.hk*
[§]Corresponding author.

measurement, computation time, and execution time. It should be noted that time delays are often the main cause for instability and poor performance of systems, and systems with time delays are much more complex than nondelay systems [25,26]. Thus, cooperative control problems of MASs with communication delays have received increasing attention recently [21,27–31].

It should also be noted that digital microprocessors become indispensable in practical applications, and therefore, cooperative control strategies for MASs tend to be implemented on digital platforms. In practice, cooperative control strategies of MASs, such as communication between neighboring agents and control law updates, are often implemented in discrete-time via a periodic sampled manner. To handle the worst scenario, the sampling period needs to be sufficiently small. Thus, there might be many unnecessary samplings in typical implementation of cooperative control strategies. Compared with the periodic sampling strategy, the event-triggered strategy executes sampling only when necessary. Thus, this strategy is able to reduce the number of samplings while maintaining the control performance [32–34]. This is particularly important for cooperative control of MASs when communication between neighboring agents is required.

In this paper, we aim to provide an overview of the recent advances in distributed coordination of MASs. It should be pointed out that the literature on distributed coordination of MASs is so huge that an exhaustive list is impossible. Instead, only a small set of the list is reviewed in this paper. Many excellent works are unfortunately left out. A block diagram illustrating the structure of this survey is shown in Fig. 1.

More specifically, this review mainly focuses on the following topics:

(1) Cooperative control of MASs with various agent dynamics. Agent dynamics under review include homogeneous and heterogeneous linear dynamics, unknown linear dynamics, and nonlinear dynamics. Cooperative control problems under review include consensus, cooperative output regulation (COR), formation control and containment control.

(2) Cooperative control of MASs with communication delays. Various results on cooperative control of MASs with bounded transmission delays and unbounded transmission delays between neighboring agents, respectively, are reviewed.

(3) Event-triggered cooperative control. Various results on event-triggered cooperative control of MASs are reviewed. Two approaches to avoid continuous monitoring of neighboring agents information are highlighted. Furthermore, the issue of avoiding Zeno behavior is also emphasized.

The rest of the paper is organized as follows. Section 2 introduces some basic graph theory and notations. Sections 3–5 review the recent advances on cooperative control of MASs with various agent dynamics, cooperative control of MASs with communication delays, and event-triggered cooperative control, respectively. Concluding remarks and future potential topics of distributed coordination of MASs are presented in Sec. 6.

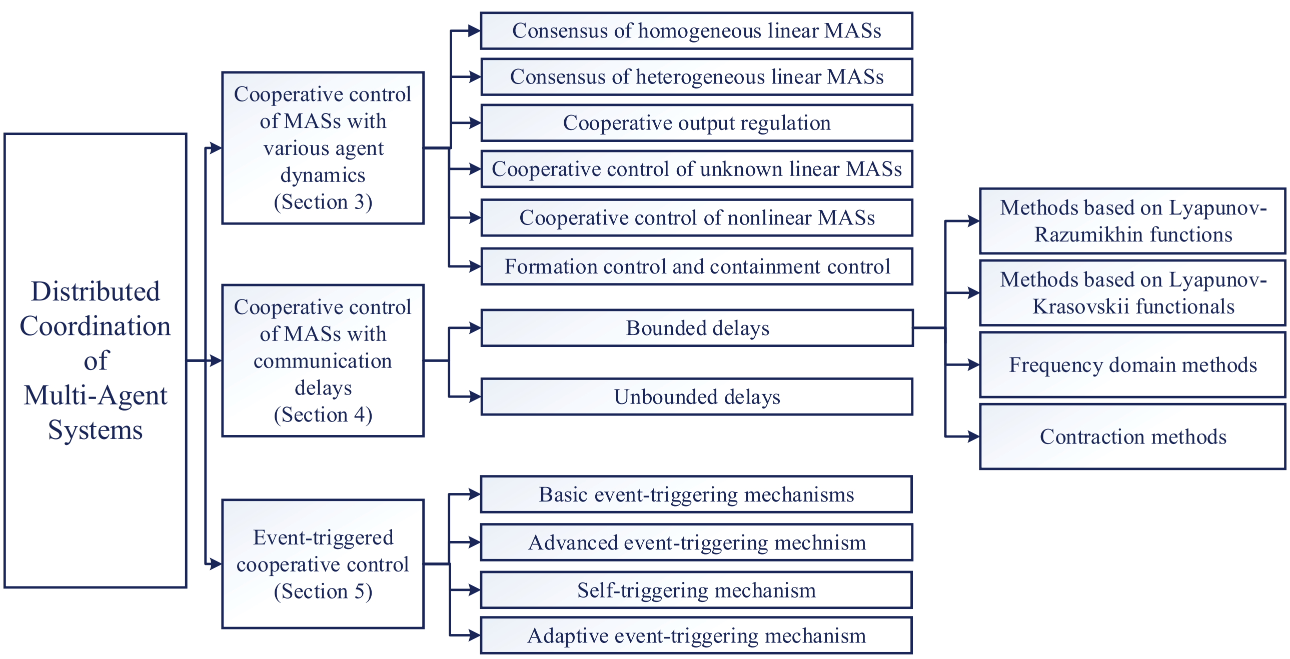

Fig. 1. Main structure of this survey.

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2. Preliminaries

Consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the vertex set $\mathcal{V} = \{1, \ldots, N\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. If $(i, j) \in \mathcal{E}$, node i can send information to node j directly, node j is called an out-neighbor of node i, and node i is called an in-neighbor of node j. The graph $\mathcal{G}$ is undirected, if and only if $(i, j) \in \mathcal{E}$ implies that $(j, i) \in \mathcal{E}$. The graph $\mathcal{G}$ has a directed spanning tree if and only if there is at least one root, which has no in-neighbors and has a directed path to every other node in $\mathcal{G}$. When the graph $\mathcal{G}$ is undirected, $\mathcal{G}$ is connected if there is a path between every pair of distinct nodes.

The adjacency matrix of $\mathcal{G}$ is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$, where $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, else $a_{ij} = 0$. The Laplacian matrix of $\mathcal{G}$ is denoted by $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$, where $l_{ij} = \{ {}^{-a_{ij},}_{\sum_{j=1}^{N} a_{ij},} \; {}^{i\neq j}_{i=j}$. $\mathcal{N}_i = \{j \in \mathcal{V}, (j, i) \in \mathcal{E}\}$ is the neighbor set of node i. For undirected graphs, $\mathcal{L}$ is symmetric and positive semi-definite, i.e. $\mathcal{L}^T = \mathcal{L} \geq 0$. For directed graphs, the Laplacian matrix $\mathcal{L}$ is not necessarily symmetric. For both directed and undirected graphs, $\mathbf{1}_N$ is an eigenvector corresponding to the eigenvalue $\lambda_1(\mathcal{G}) = 0$, i.e. $\mathcal{L}\mathbf{1}_N = 0$.

If there exists an exosystem, the network topology among an exosystem and N agents is modeled by a graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ with node 0 associated with the exosystem and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. Define $\mathcal{D} = \mathrm{diag}\{a_{10}, \ldots, a_{N0}\}$, where $a_{i0} = 1$, if agent i has access to information from the exosystem, and $a_{i0} = 0$ otherwise. The matrix $\mathcal{H} = \mathcal{L} + \mathcal{D}$ represents the connectedness of the graph $\bar{\mathcal{G}}$. Node 0 is said to be globally reachable in $\bar{\mathcal{G}}$ if there is a path from every agent to node 0.

Given a set of graphs $\mathcal{G}_\sigma = (\mathcal{V}, \mathcal{E}_\sigma)$, $\sigma = 1, \ldots, \rho$. The graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, $\mathcal{E} = \bigcup_{\sigma=1}^{\rho} \mathcal{E}_\sigma$ is regarded as the union of $\mathcal{G}_\sigma$, $\sigma = 1, \ldots, \rho$, i.e. $\mathcal{G} = \bigcup_{\sigma=1}^{\rho} \mathcal{G}_\sigma$. $\sigma(t)$ is the piecewise constant switching signal at a sequence of switching instants $\{t_l : l \in \mathbb{N}^+\}$ satisfying $t_l - t_{l-1} \geq \iota > 0$, i.e. $\sigma(t) = p$, for $t \in [t_{l-1}, t_l)$ and some integer $1 \leq p \leq \rho$. Define a time-varying graph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$ with $\mathcal{E}_{\sigma(t)} \subseteq \mathcal{V} \times \mathcal{V}$ for all $t \geq 0$. For any $t \geq 0$, $s > 0$, let $\mathcal{G}([t, t+s)) = \bigcup_{t_l \in [t, t+s)} \mathcal{G}_{\sigma(t)}$. The time-varying graph $\mathcal{G}_{\sigma(t)}$ is said to be uniformly connected if the union graph $\mathcal{G}([t, t+s))$ is connected.

Notations: Denote $\mathbb{R}^{m\times n}$ as the set of $m \times n$ real matrices. $\mathbb{N}^+$ represents the set of all positive integers. $\bar{\mathbb{C}}_+$ denotes the closed right-half complex plane. The notations $\|\cdot\|$ and $\otimes$ represent Euclidean norm and Kronecker product, respectively. The symbol $\det(\cdot)$ denotes the determinant of a matrix, $\arg(\cdot)$ is the phase angle of a complex number, and $\lambda_{\max}(\cdot)$ represents the maximum eigenvalue of a matrix. $y(t) = G(s)[u(t)]$ represents the output of a system characterized by a dynamic operator $G(s)$ with input $u(t)$.

3. Cooperative Control of MASs with Various Agent Dynamics

In this section, recent advances on cooperative control of MASs with various agent dynamics will be reviewed.

3.1. *Consensus of homogeneous linear MASs*

Consensus problems of MASs have been extensively studied in the past decades [35–54]. One reason is that many other cooperative control problems including COR, containment control, and formation control can be viewed as extensions of consensus problems or can be solved by similar consensus protocols.

In early works, many researchers focus on the consensus problem of homogeneous MASs with simple agent dynamics such as single- or double-integrator dynamics, see, for example, [41–46]. On the basis of these works, the consensus problem of homogeneous MASs with general linear dynamics has been studied [47–50].

Consider a group of N agents with general linear dynamics, where single- and double-integrator can be regarded as its special cases. The dynamics of the agents are described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \ldots, N, \tag{1}$$

where $x_i(t)$, $y_i(t)$ and $u_i(t)$ are the state, output and control input of agent i, respectively. It is assumed that the pair (A, B) is stabilizable. The graph $\mathcal{G}$ represents the underlying communication graph between neighboring agents.

The state consensus problem can be stated as follows: design a distributed control law $u_i(t)$ such that for any initial condition, the states of the agents satisfy

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \quad i, j = 1, \ldots, N. \tag{2}$$

In this case, the state consensus is said to be achieved.

In order to solve the consensus problem, the following distributed consensus control protocol is commonly used:

$$u_i(t) = \mu K \sum_{j=1}^{N} a_{ij}(t)(x_i(t) - x_j(t)), \tag{3}$$

where μ is the coupling weight, and K is the controller gain matrix.

When the communication topology is fixed, the closed-loop system consisting of the MAS (1) and the controller (3) can be described by

$$\dot{x}(t) = (I_N \otimes A + \mu\mathcal{L} \otimes BK)x(t), \tag{4}$$

where $x(t) = [x_1^T(t), \ldots, x_N^T(t)]^T$. Define $\delta(t) = (\hat{M} \otimes I_n)x(t)$, where $\hat{M} = I_N - \mathbf{1}r^T$ and $r^T = [r_1, \ldots, r_N] \in \mathbb{R}^{1\times N}$ satisfies $r^T\mathcal{L} = 0$ and $r^T\mathbf{1} = 1$. Then, it follows from (4) that

$$\dot{\delta}(t) = (I_N \otimes A - \mu\mathcal{L} \otimes BK)\delta(t). \tag{5}$$

According to the definition of r, one can obtain that $\hat{M}$ has a simple eigenvalue 0 with $\hat{M}\mathbf{1} = 0$, and $N-1$ multiple eigenvalue 1. Thus, $\delta = 0$ if and only if $x_1 = \cdots = x_N$.

In what follows, we present the results on stability analysis of the closed-loop MAS suggested in [51,52]. Let $Y \in \mathbb{R}^{N\times(N-1)}$, $W \in \mathbb{R}^{(N-1)\times N}$, and $T \in \mathbb{R}^{N\times N}$ be such that

$$T = [1, Y], \quad T^{-1} = \begin{bmatrix} r^T \\ W \end{bmatrix}, \quad T^{-1}\mathcal{L}T = J = \begin{bmatrix} 0 & 0 \\ 0 & \Delta \end{bmatrix}, \quad (6)$$

where the diagonal entries of the upper triangular Δ are the nonzero eigenvalues of $\mathcal{L}$. Define

$$\varepsilon(t) = (T^{-1} \otimes I_{2n})\delta(t), \quad (7)$$

where $\varepsilon(t) = [\varepsilon_1^T(t), \ldots, \varepsilon_N^T(t)]^T$. Then, (5) can be represented in terms of $\varepsilon(t)$ as follows:

$$\dot{\varepsilon}(t) = (I_N \otimes A + \mu J \otimes BK)\varepsilon(t). \quad (8)$$

On the other hand, it can be seen that

$$\varepsilon_1(t) = (r^T \otimes I_{2n})\delta(t) \equiv 0. \quad (9)$$

Hence, for $i = 2, \ldots, N$, $\varepsilon_i(t)$ converges asymptotically to zero if and only if $A + \mu\lambda_i BK$ is Hurwitz. Then, the following result is readily obtained.

Theorem 3.1 ([51]). *Consider MAS* (1) *with the communication topology $\mathcal{G}$ that contains a directed spanning tree. The distributed controller* (3) *solves the consensus problem if and only if there exists K such that*

$$A + \mu\lambda_i BK, \quad (10)$$

is Hurwitz for $i = 2, \ldots, N$, where λ_i is the nonzero eigenvalue of its Laplacian matrix.

Remark 3.2. It should be noted that the undirected graph can be regarded as a special case of the directed graph, and the eigenvalues of the Laplacian matrix in this case are of real values.

It is in general difficult to guarantee condition (10) due to the coupling between the local agent feedback control design and the graph topological properties. To overcome this difficulty, a Riccati design approach is proposed in [52], which provides the following design method for feedback matrix K in protocol (3) such that K is independent of the graph topology.

Theorem 3.3 ([52]). *Consider MAS* (1) *with the communication graph $\mathcal{G}$ that contains a directed spanning tree. Design the distributed control protocol* (3) *with*

$$K = -R^{-1}B^TP, \quad (11)$$

where $P > 0$ is a solution to the following algebraic Riccati equation (*ARE*):

$$0 = AP + PA^T + Q - PBR^{-1}B^TP, \quad (12)$$

with R and Q being positive definite matrices. Then, consensus will be achieved asymptotically if the coupling gain satisfies

$$\mu \geq \frac{1}{\min\limits_{i=2,\ldots,N} \mathrm{Re}(\lambda_i)}. \quad (13)$$

Proof. Let the nonzero eigenvalues of $\mathcal{L}$ be $\lambda_i = \alpha_i + j\beta_i$, where $\alpha_i, \beta_i \in \mathbb{R}$. Considering (11) and (12), one can obtain that

$$\begin{aligned}&(A - \mu\lambda_i BK)^TP + P(A - \mu\lambda_i BK) \\ &\quad = -Q - (2\mu\alpha_i - 1)K^TRK. \quad (14)\end{aligned}$$

Since $P > 0$ and $Q > 0$, by Lyapunov theory, the matrix $A - \mu\lambda_i BK$ is Hurwitz if $\mu \geq \frac{1}{\min_{i=2,\ldots,N}\mathrm{Re}(\lambda_i)}$. Then, the proof is completed by invoking the result of Theorem 3.1. □

When the communication topology between neighboring agents is switched but undirected, under the distributed controller (3) with $K = B^TP$, the closed-loop system of agent i can be given by

$$\dot{x}_i(t) = Ax_i(t) + BB^TP\sum_{j=1}^{N} a_{ij}(t)(x_j(t) - x_i(t)). \quad (15)$$

Let $x_c(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t)$. Since the network graph is undirected, from (15), one can obtain that

$$\dot{x}_c(t) = Ax_c(t). \quad (16)$$

Then, decompose $x_i(t)$ into $x_i(t) = x_c(t) + d_i(t)$, where $d_i(t)$ is called the disagreement vector. If $\lim_{t\to\infty} d_i(t) = 0$, $x_i(t)$ will converge to $x_c(t)$, and consensus can be achieved. Define $d(t) = [d_1^T(t), \ldots, d_N^T(t)]^T$. The disagreement vector $d(t)$ satisfies

$$\dot{d}(t) = (I_N \otimes A - \mathcal{L}_{\sigma(t)} \otimes (BB^TP))d(t). \quad (17)$$

Choose a Lyapunov function candidate $V(d(t)) = \frac{1}{2}d^T(t)d(t)$. To analyze the switched system (17), a modified Barbalat lemma is needed, which is shown as follows.

Lemma 3.4 ([50]). *Given the sequence $\{t_l\}$ with $t_0 = 0$, $t_l - t_{l-1} \geq \iota > 0$, $l \in \mathbb{N}^+$. If $V(t)$, $t \in [0, \infty)$ is continuous and satisfies*

(1) *$V(t)$ is lower bounded;*
(2) *$\dot{V}(t)$, $[t_{l-1}, t_l)$, $l \in \mathbb{N}^+$ is nonpositive and differentiable;*
(3) *there exists a constant $c > 0$ such that*

$$\sup_{t_{l-1}\leq t<t_l, l\in\mathbb{N}^+} |\ddot{V}(t)| \leq c, \quad (18)$$

then $\lim_{t\to\infty}\dot{V}(t) = 0$.

By using Lemma 3.4, one can obtain the following result under the following two assumptions.

Assumption 3.5. (A, B) is controllable and there exists a matrix $P > 0$ satisfying $PA + A^T P \leq 0$.

Assumption 3.6. The time-varying graph $\mathcal{G}_{\sigma(t)}$ is uniformly connected and undirected for any $t \geq 0$.

Theorem 3.7 ([50]). *Under Assumptions* 3.5 *and* 3.6, *MAS* (1) *with the distributed controller* (3) *achieves consensus asymptotically.*

It should be noted that in those earlier results, when designing the coupling weight μ, the eigenvalues of the Laplacian matrix $\mathcal{L}$ need to be known. Thus, the distributed control protocol (3) depends on the global graph information. However, in large-scale systems, it is difficult to obtain those global graph information. To address this issue, a fully distributed consensus controller is proposed in [53], described as follows:

$$\begin{aligned} u_i(t) &= c_i(t) f_i(z_i^T(t) P^{-1} z_i(t)) K z_i(t), \\ \dot{c}_i(t) &= z_i^T(t) \Gamma z_i(t), \\ z_i(t) &= \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)), \end{aligned} \tag{19}$$

where $c_i(t)$ is the adaptive coupling weight, which is used to avoid the knowledge of global graph information. $f_i(\cdot)$ is a smooth and monotonically increasing function. Now, we are ready to present a consensus result based on the following quadratic Lyapunov function candidate:

$$V(t) = \sum_{i=1}^{N} \frac{c_i(t) q_i}{2} \int_0^{z_i^T(t) P^{-1} z_i(t)} f_i(s) ds + \frac{\hat{\lambda}_0}{24} \sum_{i=1}^{N} \bar{c}_i^2(t), \tag{20}$$

where $\bar{c}_i(t) = c_i(t) - \alpha$, $\alpha > 0$.

Theorem 3.8 ([53]). *Suppose that the graph $\mathcal{G}$ contains a directed spanning tree. Let $K = -B^T P$, $\Gamma = P^{-1} B B^T P^{-1}$, and $f_i(z_i^T(t) P^{-1} z_i(t)) = (1 + z_i^T(t) P^{-1} z_i(t))^3$, where $P > 0$ satisfies*

$$AP + PA^T - 2BB^T < 0. \tag{21}$$

The consensus problem of MAS (1) can be solved by the adaptive control protocol (19). Moreover, the adaptive parameters $c_i(t)$, $i = 1, \ldots, N$ converge to some finite constants.

3.2. *Consensus of heterogeneous linear MASs*

In many real applications, agent dynamics and even their state dimensions might be different. Such MASs are referred to as heterogeneous MASs. In this case, state consensus becomes meaningless, and thus, output consensus problems are usually studied. For heterogeneous MASs, output consensus is said to be achieved if all the agents' outputs reach a common value or vector. In the case of leader-following MASs, leader-following output consensus is said to be achieved if all the agents' outputs synchronize with the output of the leader.

Consider a leaderless heterogeneous MAS, which can be described as follows:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t), \\ y_i(t) &= C_i x_i(t), \ i = 1, \ldots, N, \end{aligned} \tag{22}$$

where (A_i, B_i) is stabilizable and (C_i, A_i) is detectable.

The leaderless output consensus is said to be achieved if

$$\lim_{t \to \infty} \|y_i(t) - y_j(t)\| = 0, \quad i, j = 1, \ldots, N. \tag{23}$$

To deal with the heterogeneity of the MAS (22) under a directed time-varying graph $\mathcal{G}_{\sigma(t)}$, a distributed controller based on an internal reference model is developed in [54], and is shown as follows:

$$\begin{aligned} u_i(t) &= K_i(\hat{x}_i(t) - \Pi_i \eta_i(t)) + \Gamma_i \eta_i(t), \\ \dot{\eta}_i(t) &= S \eta_i(t) + \sum_{j=1}^{N} a_{ij}(t)(\eta_j(t) - \eta_i(t)), \\ \dot{\hat{x}}_i(t) &= A_i \hat{x}_i(t) + B_i u_i(t) + H_i(\hat{y}_i(t) - y_i(t)), \end{aligned} \tag{24}$$

where $\hat{y}_i(t) = C_i \hat{x}_i(t)$, $\eta_i(t)$ is the state of the internal reference model, S, K_i, Π_i, Γ_i and H_i are all design matrices.

Define the tracking errors $\tilde{x}_i(t) = x_i(t) - \Pi_i \eta_i(t)$ and the observer errors $\bar{x}_i(t) = x_i(t) - \hat{x}_i(t)$. Then, the closed-loop system can be expressed as follows:

$$\begin{aligned} \dot{\tilde{x}}_i(t) &= (A_i + B_i K_i)\tilde{x}_i - BK\bar{x}_i(t) \\ &\quad - \Pi_i \sum_{j=1}^{N} a_{ij}(t)(\eta_j(t) - \eta_i(t)), \\ \dot{\bar{x}}_i(t) &= (A_i + H_i C_i)\bar{x}_i(t), \\ \dot{\eta}_i(t) &= S\eta_i(t) + \sum_{j=1}^{N} a_{ij}(t)(\eta_j(t) - \eta_i(t)). \end{aligned} \tag{25}$$

One then has the following result.

Theorem 3.9 ([54]). *Suppose that the graph $\mathcal{G}_{\sigma(t)}$ is uniformly connected. Choose K_i and H_i such that $A_i + B_i K_i$ and $A_i + H_i C_i$ are Hurwitz. Assume there exist matrices S and R, where $\sigma(S) \subset j\mathbb{R}$, $\sigma(S)$ is the spectrum of S and (S, R) is observable, and matrices Π_i and Γ_i, $i = 1, \ldots, N$ satisfying*

$$\begin{aligned} A_i \Pi_i + B_i \Gamma_i &= \Pi_i S, \\ C_i \Pi_i &= R, \quad i = 1, \ldots, N. \end{aligned} \tag{26}$$

Then, under distributed controller (24), the outputs of heterogeneous MAS (22) uniformly exponentially converge to an output trajectory of the virtual exosystem defined by the pair (S, R), that is, $\dot{v}(t) = Sv(t)$, $y_0(t) = Rv(t)$.

3.3. *Cooperative output regulation*

COR can be viewed as an extension of consensus of MASs. The objective is to design a distributed controller such that each agent can track a class of reference inputs and at the same time reject a class of external disturbances while maintaining the closed-loop system stability. In general, the COR problem is more challenging than the leader-following consensus problem since it needs to achieve not only trajectory tracking but also disturbance rejection. The commonly used methods for solving the COR problem of MASs are the feedforward design approach and the internal model approach.

The feedforward design approach to the COR problem is developed in [55,56]. Consider the following heterogeneous MAS:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + E_i v(t), \\ e_{yi}(t) &= C_i x_i(t) + D_i u_i(t) + F_i v(t), \\ y_{mi}(t) &= C_{mi} x_i(t) + D_{mi} u_i(t) + F_{mi} v(t), \end{aligned} \tag{27}$$

where $e_{yi}(t) \in \mathbb{R}^{p_i}$ and $y_{mi}(t) \in \mathbb{R}^{p_{mi}}$ are the regulated output and measurement output of agent i, respectively, the pair (A_i, B_i) is stabilizable, and $v(t) \in \mathbb{R}^q$ is the external signal representing the reference signal to be tracked and/or the disturbance to be rejected, which is generated by the following exosystem:

$$\dot{v}(t) = A_0 v(t), \tag{28}$$

where A_0 is the system matrix of the exosystem.

The graph $\bar{\mathcal{G}}$ represents communication graph between N agents and an exosystem.

The COR is said to be achieved if

(1) The overall closed-loop system excluding the exosystem is asymptotically stable.

(2) For any $x_i(0)$ and $v(0)$, $\lim_{t\to\infty} e_{yi}(t) = 0$.

The following assumptions are made to solve the COR problem.

Assumption 3.10. A_0 has no eigenvalues with negative real parts.

Assumption 3.11. The following linear matrix equations:

$$\begin{aligned} X_i A_0 &= A_i X_i + B_i U_i + E_i, \\ 0 &= C_i X_i + D_i U_i + F_i, \end{aligned} \tag{29}$$

have a solution pair (X_i, U_i).

Assumption 3.12. The graph $\bar{\mathcal{G}}$ contains a directed spanning tree with the exosystem as the root.

To deal with the COR problem of the MAS (27)*, the distributed controller based on the feedforward design approach is given as follows* [*55*]:

$$\begin{aligned} u_i(t) &= K_{1i} x_i(t) + K_{2i} \hat{v}_i(t), \\ \dot{\hat{v}}_i(t) &= A_0 \hat{v}_i(t) + \mu \left(\sum_{j \in \mathcal{N}_i} a_{ij} (\hat{v}_j(t) - \hat{v}_i(t)) + a_{i0} (v(t) - \hat{v}_i(t)) \right), \end{aligned} \tag{30}$$

where K_{1i} and K_{2i} are the gain matrices, and $\hat{v}_i(t)$ is the state of the distributed observer. Then, the following result can be obtained.

Theorem 3.13 ([55]). *Suppose Assumptions* 3.10–3.12 *hold. Choose K_{1i} such that $A_i + B_i K_{1i}$ is Hurwitz, and $K_{2i} = U_i - K_{1i} X_i$. Under the distributed controller* (30) *with sufficiently large positive number μ, the COR problem is solved.*

It should be noted that the feedforward design approach to cooperative control of heterogeneous MASs described in (30) cannot be utilized to deal with the MASs subject to parameter uncertainties. In this case, the internal model principle can be adopted to solve the cooperative control problem of heterogeneous MASs with structural agent uncertainties [19,57,58]. Consider an MAS with structural agent uncertainties as follows:

$$\begin{aligned} \dot{x}_i(t) &= (A_i + \delta_i A) x_i(t) + (B_i + \delta_i B) u_i(t) \\ &\quad + (E_i + \delta_i E) v(t), \\ y_i(t) &= (C_i + \delta_i C) x_i(t), \quad i = 1, \ldots, N, \end{aligned} \tag{31}$$

where $v(t)$ is the state of the exosystem defined in (28), $\delta_i A$, $\delta_i B$, $\delta_i C$ and $\delta_i E$ are the perturbations of the agent matrices. The regulated outputs are defined as

$$e_{yi}(t) = y_i(t) - y_0(t), \quad i = 1, \ldots, N, \tag{32}$$

where $y_0(t) = Fv(t)$ for some matrix F and the pair (A_0, F) is detectable.

Then, the robust COR is said to be achieved if

(1) The nominal closed-loop system excluding the exosystem is asymptotically stable.

(2) If the perturbations are in the open neighborhood of the origin, for any $x_i(0)$ and $v(0)$, $\lim_{t\to\infty} e_{yi}(t) = 0$.

The following assumptions are standard to solve the problem.

Assumption 3.14. (A_i, B_i) is stabilizable.

Assumption 3.15. For all $\lambda \in \sigma(A_0)$,

$$\operatorname{rank} \begin{pmatrix} A_i - \lambda I & B_i \\ C_i & 0 \end{pmatrix} = n_i + p_i. \tag{33}$$

Let the informed agents that have direct access to the exosystem be denoted by $i = 1, \ldots, M$ and the uninformed agents be denoted by $i = M+1, \ldots, N$. The distributed controller is designed as follows [58]:

$$\begin{aligned}
u_i(t) &= K_{xi}x_i(t) + K_{\zeta i}\zeta_i(t),\\
\dot{\zeta}_i(t) &= G_1\zeta_i(t) + G_2(C_i x_i + D_i\hat{v}_i(t)),\\
\dot{\hat{v}}_i(t) &= A_0\hat{v}_i(t) + L(F\hat{v}_i(t) - y_0(t)), \quad i = 1, \ldots, M,\\
\dot{\hat{v}}_i(t) &= A_0\hat{v}_i(t) - (d_i(t) + f_{1i}(t))z_{vi}(t), \quad i = M+1, \ldots, N\\
\dot{d}_i(t) &= z_{vi}^T(t)\Gamma z_{vi}(t),\\
z_{vi}(t) &= \sum_{j=1}^{N} a_{ij}(\hat{v}_i(t) - \hat{v}_j(t)),\\
f_{1i}(t) &= z_{vi}^T(t)Pz_{vi}(t),
\end{aligned} \tag{34}$$

where $\zeta_i(t)$ is the state of the internal model, K_{xi}, $K_{\zeta i}$, L, Γ and P are the gain matrices, and (G_1, G_2) is the minimum p-copy internal model defined in [19]. Then, one can obtain the following result.

Theorem 3.16 ([58]). *Suppose Assumptions* 3.10*,* 3.12*,* 3.14 *and* 3.15 *hold. Choose the pair* (K_{xi}, K_{zi}) *such that* $\left[\begin{smallmatrix} A_i + B_iK_{zi} & B_iK_{zi} \\ G_2C_i & G_1 \end{smallmatrix}\right]$*,* $i = 1, \ldots, N$ *are Hurwitz, and* L *such that* $A_0 + LF$ *is Hurwitz. Let* $\Gamma = P^2$*, where* $P > 0$ *is a solution to the following:*

$$A_0^T P + PA_0 + I - P^2 = 0. \tag{35}$$

Then, the robust COR problem is solved by the distributed controller (34).

3.4. *Cooperative control of unknown linear MASs*

In many real-world systems, the system parameters might be difficult to obtain or slowly time-varying due to various reasons. In this case, it is more appropriate to consider those dynamical systems with unknown parameters. To deal with unknown parameters, many adaptive control approaches, such as model reference adaptive control approaches and adaptive pole-placement control approaches, have been well developed for single dynamical systems [59–61]. Recently, some results have been reported on cooperative adaptive control of MASs with unknown parameters, see [62–66] and the references therein. The authors in [62–64] propose distributed adaptive control strategies for unknown MASs with relative degree one. By developing an adaptive backstepping algorithm, the authors in [65] study the COR problem of heterogeneous MASs with arbitrary but uniform relative degree. To remove the limitation of uniform relative degree, the authors in [67] develop a novel design framework based on distributed observers and distributed model reference adaptive control approach.

Consider a class of heterogeneous MASs (22), where A_i, B_i and C_i are unknown system matrices. The transfer function of (22) can be expressed as follows:

$$G_{pi}(s) = C_i(sI - A_i)^{-1}B_i = k_{pi}\frac{Z_{pi}(s)}{R_{pi}(s)}, \tag{36}$$

where k_{pi} is the high-frequency gain, $Z_{pi}(s)$ and $R_{pi}(s)$ are unknown monic polynomials of degree m_{pi} and n_i, respectively, and $m_{pi} < n_i$. The leader system is described by

$$\begin{aligned}
\dot{v}(t) &= A_0v(t),\\
y_0(t) &= Fv(t),
\end{aligned} \tag{37}$$

where $v(t)$ and $y_0(t)$ are the state and output of the leader respectively, and A_0 and F are known matrices. The following assumptions are made.

Assumption 3.17. $Z_{pi}(s)$, $i = 1, \ldots, N$ are monic Huwriz polynomials, that is, the agents are of minimum phase.

Assumption 3.18. The relative degrees $r_{pi} = n_i - m_{pi}$, $i = 1, \ldots, N$ are known.

Assumption 3.19. The graph $\mathcal{G}$ between N agents is undirected and the leader is globally reachable in $\bar{\mathcal{G}}$.

Assumption 3.20. All the eigenvalues of A_0 are semi-simple with zero real parts.

A distributed adaptive control design approach is developed based on a distributed observer and a model reference adaptive controller in [67]. In what follows, the description relevant to the event-triggering mechanism addressed in [67] will be left out for better illustration. The distributed observer for agent i is the same as (30), rewritten as follows:

$$\dot{\hat{v}}_i(t) = A_0\hat{v}_i(t) - \mu\sum_{j=1}^{N} a_{ij}(\hat{v}_i(t) - \hat{v}_j(t)) - a_{i0}(\hat{v}_i(t) - v(t)), \tag{38}$$

where μ is a large enough positive constant. Based on the distributed observer, a reference model of agent i is designed as

$$\begin{aligned}
\dot{x}_{mi}(t) &= A_{mi}x_{mi}(t) + B_{mi}K_{mi}\hat{v}_i(t),\\
y_{mi}(t) &= C_{mi}x_{mi}(t),
\end{aligned} \tag{39}$$

where $x_{mi}(t)$ and $y_{mi}(t)$ are the state and output of the reference model for agent i, respectively. A_{mi}, B_{mi}, C_{mi} and K_{mi} are all design matrices. Then, the model reference adaptive controller is designed as

$$\begin{aligned}
u_i(t) &= \theta_i^T(t)w_i(t),\\
\theta_i(t) &= [\theta_{1i}^T(t), \theta_{2i}^T(t), \theta_{3i}^T(t), \theta_{4i}^T(t)]^T,\\
w_i(t) &= [w_{1i}^T(t), w_{2i}^T(t), y_i(t), K_{mi}\hat{v}_i(t)]^T,\\
\dot{w}_{1i}(t) &= \Lambda_i w_{1i}(t) + B_{\lambda i}u_i(t),\\
\dot{w}_{2i}(t) &= \Lambda_i w_{2i}(t) + B_{\lambda i}y_i(t),
\end{aligned} \tag{40}$$

where $(\Lambda_i, B_{\lambda i})$ is a controllable pair, $\theta_i(t)$ is the adaptive controller gain and the adaptation law is designed as follows:

$$\begin{aligned}
\dot{\theta}_i(t) &= \Gamma_i e_{0i}(t)\phi_i(t),\\
\dot{\rho}_i(t) &= \gamma_i e_{0i}(t)(\theta_i^T(t)\phi_i(t) + u_{fi}(t)),\\
e_{0i}(t) &= \frac{y_i(t) - y_{mi}(t) - \rho_i(t)(\theta_i^T(t)\phi_i(t) + u_{fi}(t))}{1 + \phi_i^T(t)\phi_i(t) + u_{fi}^2(t)}, \quad (41)\\
\phi_i(t) &= -G_{mi}(s)[w_i(t)],\\
u_{fi}(t) &= G_{mi}(s)[u_i(t)],
\end{aligned}$$

with the scalar $\gamma_i > 0$, the matrix $\Gamma_i > 0$, and $e_{0i}(t)$ being the estimation error. Then, the following result can be obtained.

Theorem 3.21 ([67]). *Suppose Assumptions* 3.17–3.20 *hold. There exist some matrices A_{mi}, B_{mi} and C_{mi} such that under a distributed adaptive controller composed of* (38) *to* (41)*, the outputs of all the agents track the output of the leader asymptotically and all the signals in the closed-loop system are bounded.*

3.5. *Cooperative control of nonlinear MASs*

In recent years, the cooperative control problem of MASs with nonlinear agent dynamics has also been investigated [68–75]. The agents with Lipschitz nonlinear dynamics can be described as follows [70–72]:

$$\dot{x}_i(t) = Ax_i(t) + f(x_i(t)) + Bu_i(t), \quad i = 1, \ldots, N, \quad (42)$$

where $f(x_i)$ is assumed to satisfy a Lipschitz condition. In [70], a controller with adaptive coupling weight is developed to achieve consensus of MAS (42) under an undirected communication graph. In [71], a distributed consensus protocol with stochastic sampling is proposed for MAS (42) with a directed communication graph. The authors in [72] study the leader-following consensus problem of MASs (42), where the agent dynamics satisfy Lipschitz conditions with time-varying gains. The consensus problems of networked Euler–Lagrange systems have also received particular attention since it can represent various nonlinear systems, such as spacecrafts, unmanned aerial vehicles, and so on. The Euler–Lagrange equation can be described as follows [73–75]:

$$M_i(x_i)\,\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i + g_i(x_i) = u_i, \quad i = 1, \ldots, N, \quad (43)$$

where $M_i(\cdot)$ is the symmetric positive definite inertia matrix, $C_i(\cdot)\dot{x}_i$ is the vector of Coriolis and centripetal torque, and $g_i(\cdot)$ is the vector of gravitational torque. The authors in [73] develop a distributed model-independent tracking controller via a sliding model approach for MASs (43) with a dynamic leader under a directed communication graph. In [74], a distributed adaptive controller based on an adaptive distributed observer is developed to solve the leader-following consensus problem of a group of uncertain Euler–Lagrange systems and a dynamic leader under switching topologies.

3.6. *Formation control and containment control*

Formation control and containment control can be recognized as extensions to the well-studied consensus control of MASs, and have also been studied widely. One of the most interesting topics in formation control is the so-called circular formation control. The objective of circular formation control is to make the orbit of multiple agents around a center while maintaining a spaced formation. According to the type of the center of the circular formation, circular formation control can be classified into two types. One is the collective circular motion, where the center is unspecified [76–85]. The other is the target-enclosing or circumnavigation, where the center is given [79,86–91]. Many studies focus on the second type which is of more practical significance. In particular, the authors in [92,93] study the case where the target is a nonholonomic agent and linear velocities of all agents are constant. In [94] and [95], the moving-target circular formation control problem of nonholonomic agents is studied with the time-varying velocity of the target available to all agents. In [96], a translational control design is developed for circular formation with the assumption that measurements in the global inertial frame and communication among the nonholonomic agents are available. In [97], a cooperative control law based on a consensus-based algorithm is proposed to stabilize nonholonomic agents to a circular formation towards the neighborhood of the source location. In [98], the moving-target enclosing problem is addressed where some of nonholonomic agents in the group have to know the target velocity.

Containment control arises when followers of MASs are driven to a specific area spanned by multiple leaders. The containment control problem of MASs with multiple leaders has received much attention due to their practical applications [99,100]. The authors in [101–103] develop containment controller for MASs with both fixed and switching topologies. An adaptive containment controller based on the sliding mode and backstepping control approaches is proposed in [104] for nonlinear MASs with input quantization. When the states of agents are not available, an observer-based containment protocol is developed to achieve containment control of MASs under a directed graph [105]. In [106], the output containment control problem of heterogeneous MASs is solved by proposing two control protocols based on the internal model principle. Besides, several approaches [107–109] are developed for containment control of discrete-time MASs.

3.7. *Remarks*

In this section, we have mainly reviewed two types of cooperative control problems of MASs, namely, consensus and cooperative output regulation. We have also briefly reviewed other two types of cooperative control problems, that is, formation control and containment control. We have covered MASs with various agent dynamics, such as homogeneous and heterogeneous linear dynamics, unknown linear dynamics, and nonlinear dynamics. It should be noted that the review in this section do not consider MASs subject to various communication constraints. However, communication constraints such as time-delays and bandwidth limits would naturally arise in networked systems and are known to have great adverse impact on the performance of the concerned systems. Therefore, considerable effort has been devoted to study of cooperative control of MASs with communication delays and event-triggered cooperative control of MASs, which will be reviewed in Secs. 4 and 5, respectively.

4. Cooperative Control of MASs with Communication Delays

Communication delays often exist in MASs when signals are transmitted between agents through a network. Therefore, in recent years, cooperative control problems of MASs with time-delays have received increasing attention [21,27–31]. A number of cooperative control protocols are developed for MASs with bounded time-delays or unbounded time-delays.

4.1. *Bounded delays*

The study of MASs with bounded delays is typically based on Lyapunov–Razumikhin functions, Lyapunov–Krasovskii functionals, the frequency domain method, or the contraction method, described as follows, respectively.

4.1.1. *Methods based on Lyapunov–Razumikhin functions*

The methods based on Lyapunov–Razumikhin functions for MASs with bounded delays are developed as shown in [110,111]. The authors in [110] consider an MAS with N second-order agents and a first-order leader subject to a bounded time-varying delay. To deal with the time-delay, a neighbor-based coupling rule is introduced as follows:

$$\begin{aligned} u_i(t) = &\sum_{j\in\mathcal{N}_i} a_{ij}(x_j(t-\tau(t)) - x_i(t-\tau(t))) \\ &+ a_{i0}(x_0(t-\tau(t)) - x_i(t-\tau(t))) + k(v_0 - v_i(t)), \end{aligned} \tag{44}$$

where $x_i(t)$ and $v_i(t)$ are the position vector and velocity vector of the agent i, respectively, $x_0(t)$ is the position vector of the leader, v_0 is the desired constant velocity, $k > 1$, and the time-varying delay $\tau(t) > 0$ is a continuously differentiable function satisfying $0 < \tau(t) < \bar{c}$. Defining $\check{x}(t) = x(t) - x_0(t)\mathbf{1}$, $\bar{v}(t) = v(t) - v_0\mathbf{1}$, $x(t) = [x_1(t), \dots, x_N(t)]^T$, $v(t) = [v_1(t), \dots, v_N(t)]^T$, the closed-loop system can be given as follows:

$$\dot{\bar{x}}_c(t) = C\bar{x}_c(t-\tau(t)) + E\bar{x}_c(t), \tag{45}$$

where $\bar{x}_c(t) = [\check{x}^T(t), \bar{v}^T(t)]^T$, $C = \begin{bmatrix} 0 & I_N \\ 0 & -kI_N \end{bmatrix}$, $E = \begin{bmatrix} 0 & 0 \\ -\mathcal{H} & 0 \end{bmatrix}$.

Choose a Lyapunov–Razumikhin function candidate $V(\bar{x}_c(t)) = \bar{x}_c^T(t)P\bar{x}_c(t)$, where

$$P = \begin{bmatrix} k\bar{P} & \bar{P} \\ \bar{P} & \bar{P} \end{bmatrix}, \quad k > 1, \tag{46}$$

is positive definite, $\bar{P} > 0$ satisfies $\bar{P}\mathcal{H} + \mathcal{H}^T\bar{P} = I_n$, and $\mathcal{H} > 0$. Then, the following result can be readily obtained.

Theorem 4.1 ([110]). *Suppose that the leader is globally reachable in $\bar{\mathcal{G}}$. For the system* (45)*, take $k > k^* = \frac{\bar{\mu}}{2\lambda_p} + 1$, where $\bar{\mu} = \lambda_{\max}(\bar{P}\mathcal{H}\mathcal{H}^T\bar{P})$, and λ_p is the smallest eigenvalue of $\bar{P}$. Then, when τ is sufficiently small, $\lim_{t\to\infty}\bar{x}_c(t) = 0$.*

4.1.2. *Methods based on Lyapunov–Krasovskii functionals*

The methods based on Lyapunov–Krasovskii functionals are developed as shown in [112]. To solve the consensus problem of single-integrator MASs with both switching topology and time-delay, a linear consensus protocol is introduced in [112] as follows:

$$u_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}(t)(x_j(t-\tau(t)) - x_i(t-\tau(t))), \tag{47}$$

where $x_i(t)$ is the state of agent i. Denote $x_c(0) = \frac{1}{N}\sum_{i=1}^N x_i(0)$, $\bar{\delta}(t) = x(t) - x_c(0)\mathbf{1}$. Then, the closed-loop system can be written as follows:

$$\dot{\bar{\delta}}(t) = -\mathcal{L}_\sigma\bar{\delta}(t-\tau(t)). \tag{48}$$

By defining a Lyapunov–Krasovskii functional candidate as follows:

$$\begin{aligned} V(t) = &\bar{\delta}^T(t)P\bar{\delta}(t) + \int_{t-\tau(t)}^t \bar{\delta}^T(s)Q\bar{\delta}(s)ds \\ &+ \int_{-\tau}^0\int_{t+\theta}^t \dot{\bar{\delta}}^T(s)R\dot{\bar{\delta}}(s)dsd\theta, \end{aligned} \tag{49}$$

where $P > 0$ and $Q > 0$, the following result can be readily obtained.

Theorem 4.2 ([112]). *Suppose the communication graph $\mathcal{G}_\sigma$ is strongly connected and balanced. Under the controller* (47)*, the single-integrator MAS achieves average* *consensus asymptotically, if time-delay* $\tau(t) < \bar{c}$, $\dot{\tau}(t) \leq \bar{c}_1 < 1$, *and there exist* $P > 0, Q > 0$, $R > 0$ *satisfying*

$$\begin{bmatrix} -P\bar{\mathcal{L}}_\sigma - \bar{\mathcal{L}}_\sigma^T P + Q & P\bar{\mathcal{L}}_\sigma U_1^T & \mathbf{0} \\ U_1\bar{\mathcal{L}}_\sigma^T P & -\dfrac{R(1-\bar{c}_1)}{\bar{c}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\bar{Q} + \dfrac{\bar{c}}{1-\bar{c}_1}\bar{\mathcal{L}}_\sigma^T U_1^T R U_1 \bar{\mathcal{L}}_\sigma \end{bmatrix} < 0, \tag{50}$$

where $U_1 \in \mathbb{R}^{n\times(n-1)}$ *is the first* $n-1$ *columns of* U, $\bar{\mathcal{L}}_\sigma = U_1^T \mathcal{L}_\sigma U_1$, U *is an orthogonal matrix satisfying* $U^T\Theta U = \left[\begin{smallmatrix} nI_{n-1} & \mathbf{0} \\ \mathbf{0} & 0 \end{smallmatrix}\right]$, *and*

$$\Theta = \begin{bmatrix} n-1 & -1 & \cdots & -1 \\ -1 & n-1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & n-1 \end{bmatrix}.$$

4.1.3. *Frequency domain methods*

Another method to deal with MASs with bounded delays is the frequency domain method. The authors in [113,114] consider homogeneous MASs with special forms of time delays, where agent dynamics are described as in (1) and each agent collects a delayed measurement described as follows:

$$\bar{z}_i(t) = \sum_{j=1}^{N} l_{ij} x_j(t-\tau). \tag{51}$$

The following distributed controller is designed [113]

$$u_i(t) = -\alpha B^T P_\varepsilon \bar{z}_i(t), \tag{52}$$

where $P_\varepsilon > 0$ is the solution to

$$A^T P_\varepsilon + P_\varepsilon A - P_\varepsilon B B^T P_\varepsilon + \varepsilon I = 0, \tag{53}$$

with $\varepsilon > 0$. Then, the closed-loop system is given as follows:

$$\dot{\varepsilon}_i(t) = A\varepsilon_i(t) - |\lambda_i|\alpha e^{j\psi_i} B B^T P_\varepsilon \varepsilon_i(t-\tau), \tag{54}$$

where $\varepsilon_i(t)$ is defined in (7), $\psi_i = \arg\lambda_i \in [-\varphi, \varphi]$, and $0 \leq \varphi < \frac{\pi}{2}$. If $\det[jwI - A + \alpha|\lambda_i| e^{j(\psi_i - w\tau)} B B^T P_\varepsilon] \neq 0$, the system (54) is asymptotically stable. Let

$$w_{\max} = \begin{cases} 0 & A \text{ is Hurwitz.} \\ \max\{w \in \mathbb{R} \mid \det(jwI - A) = 0\} & \text{otherwise.} \end{cases} \tag{55}$$

Then, the following result can be obtained.

Theorem 4.3 ([113]). *Suppose the graph* $\mathcal{G}$ *contains a directed spanning tree. Given* $\bar{\tau}$ *satisfying*

$$\bar{\tau} w_{\max} < \frac{\pi}{2} - \varphi, \tag{56}$$

there exist $\alpha > 0$ *and* $\varepsilon^* > 0$ *such that the MAS* (1) *with controller* (52) *achieve consensus asymptotically for any* $\tau \in [0, \bar{\tau}]$ *and* $\varepsilon \in (0, \varepsilon^*]$.

4.1.4. *Contraction methods*

Recently, the consensus problem of heterogeneous linear MASs with three most common types of delays is investigated in [115], and a contraction method is adopted. Consider the heterogeneous linear MAS (22) with time delays. Assume that (A_i, B_i) is stabilizable and (C_i, A_i) is detectable. To achieve consensus, the following controller is designed:

$$\begin{aligned} u_i(t) &= K_{1i}\hat{x}_i(t) + K_{2i}\eta_i(t), \\ \dot{\hat{x}}_i(t) &= A_i\hat{x}_i(t) + B_i u_i(t) - L_i(\hat{y}_i(t) - y_i(t)), \\ \dot{\eta}_i(t) &= S\eta_i(t) - \mu\sum_{j=1}^{N}\frac{a_{ij}}{d_i}(\eta_i(t) - T_{ij}(S, \eta_j, \tau_{ij})), \end{aligned} \tag{57}$$

where $d_i = \sum_{j=1}^{N} a_{ij}$, $T_{ij}(S, \eta_j, \tau_{ij})$ is a delay operator defined as follows:

(1) for constant delays, $T_{ij}(S, \eta_j, \tau_{ij}) = e^{S\tau_{ij}}\eta_j(t - \tau_{ij})$, $\tau_{ij} \in [0, T]$;
(2) for time-varying delays, $T_{ij}(S, \eta_j, \tau_{ij}) = e^{S\tau_{ij}(t)}\eta_j(t - \tau_{ij}(t))$, where τ_{ij} is piecewise continuous and has a dwell time $\tau_D > 0$;
(3) for distributed delays, $T_{ij}(S, \eta_j, \tau_{ij}) = \int_0^T \tau_{ij}(\theta) e^{S\theta} \times \eta_j(t-\theta) d\theta$, with delay kernel τ_{ij} satisfying $\tau_{ij}(\theta) \geq 0$, $\theta \in [0, T]$ and $\int_0^T \tau_{ij}(\theta) d\theta = 1$.

Then one can obtain the following result.

Theorem 4.4 ([115]). *Suppose the communication graph* $\mathcal{G}$ *contains a directed spanning tree, there exist matrices* (S, R) *such that* $\sigma(S) \subset \bar{\mathbb{C}}_+$, $\lambda = \max_{\lambda(S)\in\sigma(S)}\{Re[\lambda(S)]\} < \beta$ *for some constant* β, $R \neq 0$, *and the regulator equation* (26) *has a solution* (Π_i, Γ_i). *Choose* K_{1i}, K_{2i}, L_i *such that* $A_i + B_i K_{1i}$, $A_i - L_i C_i$ *are Hurwitz and* $K_{2i} = \Gamma_i - K_{1i}\Pi_i$. *Under the controller* (57), *the heterogeneous MAS* (22) *achieves consensus exponentially, and there exists a constant vector* ξ_0 *such that* $\lim_{t\to+\infty}\|y_i(t) - \mathrm{Re}^{St}\xi_0\| = 0$.

In addition, the consensus problem of discrete-time MASs with time delays has also been investigated [116–120]. The authors in [116] consider the consensus problem of single integrator MASs with constant transmission and input delays. In [117], a consensus control strategy is developed for single integrator MASs with time-varying delays. The authors in [118] consider the second-order MASs with constant communication delays and dynamically changing topologies. In [119], the consensus problem of linear MASs with a constant communication delay is discussed. Recently, the authors in [120] consider heterogeneous linear MASs with time-varying communication delays and constant input and output delays.

4.2. *Unbounded delays*

More recently, there are some works addressing cooperative control of MASs with unbounded delays. For instance, in [121,122], MASs with gamma-type distributed unbounded delays are used to model the car following systems. In [123], MASs with gamma-type distributed unbounded delays are used to model multiple coupled oscillators. In addition, single integrator MASs with time-varying unbounded delays are considered in [124,125]. Cooperative control of homogeneous linear MASs with unbounded transmission delays and fixed topology is studied in [126].

Consider a homogeneous general linear MAS (1), where all eigenvalues of A are assumed to be on the imaginary axis, (A, B) is controllable, and the communication graph between neighboring agents contains a directed spanning tree. The unbounded distributed transmission delay between neighboring agents i and j is described by a delay kernel τ_{ij}, where $\int_0^{+\infty}\tau_{ij}(z)dz = 1$ and the following assumption is satisfied.

Assumption 4.5. There exist a nonincreasing non-negative and Lebesgue integrable function p and a constant $v > 0$ such that

$$\begin{aligned} &\tau_{ij}(z) \le e^{-vz}p(z), \\ &\int_0^{+\infty} p(z)|(x_i(z) - x_j(z))|dz < +\infty. \end{aligned} \tag{58}$$

To solve the consensus problem of homogeneous general linear MASs with unbounded distributed transmission delays, the following distributed low gain controller is proposed [126]:

$$u_i(t) = -\mu B^T P \sum_{j=1}^{N} a_{ij} \int_0^{+\infty} k_{ij}(z) e^{Az}(x_i(t-z) - x_j(t-z))dz, \tag{59}$$

where P is the solution to the following parametric ARE with $\varepsilon > 0$,

$$A^T P + PA - PBB^T P = -\varepsilon P. \tag{60}$$

Denoting $\xi(t) = [(x_1(t) - x_N(t))^T, \ldots, (x_{N-1}(t) - x_N(t))^T]^T$, the closed-loop system can be given by

$$\dot{\xi}(t) = (I_N \otimes A)\xi(t) - \int_0^{+\infty} \mu\phi(\eta) \otimes BB^T P e^{A\eta}\xi(t-\eta)d\eta. \tag{61}$$

The corresponding characteristic matrix is

$$\begin{aligned} H(s) = I \otimes (sI - A) + \int_0^{+\infty} \mu\Phi(\eta) \\ \otimes BB^T P e^{-(sI-A)\eta} d\eta. \end{aligned} \tag{62}$$

According to the low gain properties newly proved in [127], one can obtain that for any $\mu > \frac{1}{Re(\lambda_2)}$, there always exists $\varepsilon^* > 0$ such that $\forall \varepsilon \in (0, \varepsilon^*]$ and for any $Re(s) \ge 0$, $\det\{H(s)\} \ne 0$. Then, via the stability theorem in [128], if $\det\{H(s)\} \ne 0$, $\forall s \in \bar{\mathbb{C}}^+$, $\lim_{t\to\infty}\xi(t) = 0$. Then, one can obtain the following result.

Theorem 4.6 ([126]). *Suppose Assumption* 4.5 *holds. There exists* $\varepsilon^* > 0$ *such that* $\forall \varepsilon \in (0, \varepsilon^*]$, *under the controller* (59) *with* $\mu > \frac{1}{\mathrm{Re}(\lambda_2)}$, *the homogeneous general linear MAS subject to unbounded distributed time delays reaches consensus asymptotically.*

On the basis of [126], the COR problem of heterogeneous linear MASs with unbounded distributed transmission delays is studied in [129]. In addition, the authors in [130] study the consensus problem of single integrator MASs with unbounded time-varying transmission delays.

4.3. *Remarks*

This section reviews the cooperative control schemes for MASs with bounded or unbounded delays. For MASs with bounded delays, four typical methods are reviewed, namely, the Lyapunov–Razumikhin functions-based methods, the Lyapunov–Krasovskii functionals-based methods, the frequency domain methods, and the contraction methods. For MASs with unbounded delays, a newly developed low gain approach is reviewed. It can be observed that the existing works mainly focus on MASs with unbounded *distributed* transmission delays under fixed communication topologies. Many open problems on distributed control of MASs with more general setting such as switching communication topologies or time-varying unbounded delays, warrant further investigation.

5. Event-Triggered Cooperative Control

Event-triggered control (ETC) is first introduced to reduce the information flow in feedback control [32–34,131–133]. It has been shown that ETC strategies can be used to reduce the number of controller update compared with traditional periodic sampling, while maintaining the system performance [32,33,134]. More recently, event-triggered strategies have been widely adopted in cooperative control of MASs to reduce the control law updates and information transmission between agents and thus to increase their operation efficiency and lifespan, see, for example, [135–172] and the references therein.

ETC strategies have been developed firstly for single-integrator, double-integrator and then homogeneous MASs [135–146]. In [142], a classical ETC strategy is proposed for consensus of single-integrator MASs. The event-triggered controller is designed as follows:

$$u_i(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}(x_i(t_k^i) - x_j(t_{k'}^j)), \tag{63}$$

where $t \in [t_k^i, t_{k+1}^i)$, $t_{k'}^j$ is the latest triggering instant of agent j. Denote the measurement error as $e_i(t) = x_i(t_k^i) - x_i(t)$. Then, the following three basic event-triggering mechanisms are proposed:

(1) The state-dependent triggering mechanism is given by

$$t_{k+1}^i = t_k^i + \inf_{t>t_k^i}\left\{t - t_k^i \middle| \|e_i(t)\|^2 \geq \frac{\sigma_i a(1 - a|\mathcal{N}_i|)}{|\mathcal{N}_i|} \times \|z_i^2(t)\|\right\}, \tag{64}$$

where $z_i(t)$ is defined in (19).

(2) The absolute triggering mechanism is given by

$$t_{k+1}^i = t_k^i + \inf_{t>t_k^i}\{t - t_k^i \,|\, \|e_i(t)\| \geq \delta\}, \tag{65}$$

where $\delta > 0$.

(3) The time-dependent triggering mechanism is given by

$$t_{k+1}^i = t_k^i + \inf_{t>t_k^i}\{t - t_k^i \,|\, \|e_i(t)\| \geq c_0 + c_1 e^{-\alpha t}\}, \tag{66}$$

where $c_0 > 0$ and $c_1 > 0$.

It is shown that under the ETC strategy (63) and the event-triggering mechanisms (64)–(66), the MAS achieves consensus asymptotically, meanwhile, the frequency of agent control actuation is decreased significantly. Though the ETC strategy proposed in [142] enjoys some advantages, it also suffers from some limitations such as (i) continuous monitoring of neighboring agent states and (ii) Zeno behavior. First, it can be observed from the event-triggering mechanism (64) that when determining the next triggering instant, continuous monitoring of neighbors' states is required. In other words, this mechanism will not be able to reduce communication load. In order to reduce data transmission, other two event-triggering mechanisms can be used. In addition, a time-dependent threshold is proposed in [143], which do not need to monitor neighbors' states continuously. Second, the ETC strategy (63) with (64) is effective for single integrator MASs. However, it fails to work when agents are of high-order integrator and general linear dynamics. More specifically, when consensus of the concerned MAS is achieved, the threshold function will approach zero but the error function may not converge to zero. Therefore, the event-triggering mechanism (64) may lead to Zeno behavior which is not desirable.

Considerable effort has been devoted to developing new distributed event-triggering mechanisms to avoid those limitations. For example, in order to tackle the second limitation, the following ETC strategy and event-triggering mechanism are proposed [144],

$$\begin{aligned} u_i(t) &= -K\sum_{j\in N_i} a_{ij}(x_i(t_k^i) - x_j(t_k^i)), \\ t_{k+1}^i &= t_k^i + \inf_{t>t_k^i}\left\{t - t_k^i \middle| d_i\|e_i(t)\| + b_i\|\tilde{e}_i(t)\| \geq \beta\sqrt{\sum_{j\in\mathcal{N}_i}\|x_j(t) - x_i(t)\|^2} + \delta_i\right\}, \end{aligned} \tag{67}$$

where $\tilde{e}_i(t) = [e_{i1}^T(t), \ldots, e_{i(i-1)}^T(t), e_{i(i+1)}^T(t), \ldots, e_{iN}^T(t)]^T$, $e_{ij}(t) = x_j(t_k^i) - x_j(t)$, $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$, $b_i = \sqrt{\sum_{j\in\mathcal{N}_i} a_{ij}^2}$, β and δ_i are positive constants. However, by using this event-triggering mechanism (67), only bounded consensus rather than perfect consensus can be achieved. To deal with this issue, the authors in [145] propose a novel ETC strategy based on the so-called combinational measurement approach. Define the following combinational measurement error:

$$e_{zi}(t) = z_i(t_k^i) - z_i(t). \tag{68}$$

The control law for agent i is proposed as follows:

$$u_i(t) = Kz_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i). \tag{69}$$

Then, the next triggering instant will be generated by

$$t_{k+1}^i = t_k^i + \inf_{t>t_k^i}\{t - t_k^i \,|\, \|e_{zi}(t)\| > \beta_i\|z_i(t)\|\} \tag{70}$$

with $\beta_i > 0$. On the basis of the combinational measurement approach developed in [145], the authors in [146] further study the consensus problem of homogeneous linear MASs. Compared with [144], the ETC strategies proposed in [145,146] achieve perfect consensus asymptotically. It should be noted that there is an error in the proof on the exclusion of Zeno behavior in [145] which has been fixed recently [173].

Then, based on the combinational measurement approach, some advanced event-triggering mechanisms are proposed [147–151]. The event-triggering mechanism with a timer is given as follows [147]:

$$t_{k+1}^i = t_k^i + \max\{\tau_k^i, \tau_i\}, \tag{71}$$

where

$$\tau_k^i = \inf_{t>t_k^i}\{t - t_k^i \,|\, \|e_{zi}(t)\| = \gamma_i\|z_i(t)\|\}, \tag{72}$$

and τ_i is the fixed timer acting as the lower bound of the inter-event times. A unified framework of time- and event-triggering strategy is given as follows [148]:

$$t_{k+1}^i = \max\left\{\inf_{t>t_k^i}\{t|f_i^1(\|e_{zi}(t)\|, \|z_i(t)\|) = 0\},\ \inf_{t>t_k^i}\{t|f_i^2(t, t_k^i, \tau_i) = 0\}\right\}, \tag{73}$$

where $f_i^1(\|e_{zi}(t)\|, \|z_i(t)\|) = \|e_{zi}(t)\| - \sigma_i\|z_i(t)\|$ and $f_i^2(t, t_k^i, \tau_i) = t - t_k^i - \tau_i$. To reduce the inter-event intervals, a dynamic triggering mechanism with an additional internal state variable is proposed in [149], and is given as follows:

$$\begin{aligned} t_{k+1}^i &= t_k^i + \inf_{t>t_k^i}\{t - t_k^i|\|e_{zi}(t)\|^2 \geq \delta_i\|z_i(t)\|^2 + \pi_i\varrho_i(t)\},\\ \dot{\varrho}_i(t) &= -\beta_i\varrho_i(t) + \theta_i(\delta_i\|z_i(t)\|^2 - \|e_{zi}(t)\|^2), \end{aligned} \tag{74}$$

where $\varrho_i(0) > 0$.

An approach to overcome the first limitation is the so-called open-loop estimation approach [139,152–154]. The event-triggered controller based on state feedback is given as follows [139,153]:

$$u_i(t) = \mu K\sum_{j\in\mathcal{N}_i}(e^{A(t-t_k^i)}x_i(t_k^i) - e^{A(t-t_{k'}^j)}x_j(t_{k'}^j)), \tag{75}$$

where $t \in [t_k^i, t_{k+1}^i)$. The measurement error based on the open-loop estimation is defined as

$$e_{xi}(t) = e^{A(t-t_k^i)}x_i(t_k^i) - x_i(t). \tag{76}$$

Then, the state-dependent triggering mechanism is designed as follows [139]:

$$\begin{aligned} t_{k+1}^i &= t_k^i + \inf_{t>t_k^i}\{t - t_k^i|e_{xi}^T(t)\Phi e_{xi}(t) > \sigma_i\hat{z}_i^T(t)\Phi\hat{z}_i(t) + \delta_i\},\\ \hat{z}_i(t) &= \sum_{j\in\mathcal{N}_i}a_{ij}(e^{A(t-t_k^i)}x_i(t_k^i) - e^{A(t-t_{k'}^j)}x_j(t_{k'}^j)), \end{aligned} \tag{77}$$

where σ_i and δ_i are positive constants. A time-dependent triggering mechanism is proposed in [153], and is given as follows:

$$t_{k+1}^i = t_k^i + \inf_{t>t_k^i}\{t - t_k^i|\|e_{xi}(t)\| > c_1e^{-\alpha t}\}, \tag{78}$$

where c_1 and α are positive constants. It can be observed that in [139,153], the next triggering instant only depends on the information of neighbors at their triggering instants, and thus, continuous monitoring can be avoided.

Another approach to overcome the first limitation is the so-called self-triggering mechanism, where the next sampling instant is predicted based on the information at the latest triggering instant and the knowledge of agent dynamics [142,147,155–161]. A self-triggering mechanism is presented in [142] to achieve consensus of single integrator MASs. The next triggering instant is generated by

$$t_{k+1}^i = t_k^i + \xi_i, \tag{79}$$

where ξ_i satisfies

$$|\rho_i|\xi_i = \sqrt{\beta_i}|P_i\xi_i + \Phi_i|, \tag{80}$$

with $\rho_i = \sum_{j\in\mathcal{N}_i}(x_i(t_k^i) - x_j(t_{k'}^j))$, $P_i = -|\mathcal{N}_i|\rho_i + \sum_{j\in\mathcal{N}_i}\rho_j$, $\Phi_i = |\mathcal{N}_i\rho_i| + \sum_{j\in\mathcal{N}_i}(\rho_j(t_k^i - t_{k'}^j))$, and $\beta_i > 0$. It has been shown in [142] that the self-triggering mechanism is more robust but will lead to more controller updates compared with the event-triggering mechanism. However, the proposed self-triggering mechanism in [142] cannot exclude Zeno behavior. To solve this problem, a novel self-triggering mechanism based on the combinational measurement approach is developed in [159] for single integrator MASs, where a fixed timer is introduced into the proposed mechanism. On the basis of [159], a self-triggering mechanism is developed in [147] for heterogeneous linear MASs.A distributed self-triggering mechanism is proposed in [174] for resilient output synchronization of heterogeneous linear MASs subject to denial-of-service (DoS) attacks. Moreover, the authors in [175] propose a team-triggering mechanism, which combines the advantages of both event-triggering and self-triggering mechanisms.

It should be noted that most of the existing works on ETC of MASs need to know the minimum nonzero eigenvalue of the corresponding Laplacian matrix λ_2 and the number of agents. In recent years, some fully distributed adaptive ETC strategies are proposed. In [162,163], two types of fully distributed ETC strategies based on node information and edge information are developed for homogeneous MASs, respectively. The fully distributed adaptive event-triggered controller based on the node information is given as follows [162]:

$$\begin{aligned} u_i(t) &= \bar{c}_i(t)K\hat{z}_i(t),\\ \dot{c}_i(t) &= e^{2\lambda t}\hat{z}_i^T(t)\Phi\hat{z}_i(t), \end{aligned} \tag{81}$$

where $\bar{c}_i(t)$ is updated to be $c_i(t)$ and broadcast to neighbors whenever $c_i(t)$ reaches previous $\bar{c}_i(t) + \kappa_i$, $\kappa_i > 0$. Then, the adaptive event-triggering mechanism is given as

$$t_{k+1}^i = t_k^i + \inf_{t>t_k^i}\{t - t_k^i|\|e_{xi}(t)\| > \gamma_i(t)\|\hat{z}_i(t)\| + \delta_i(t)e^{-\alpha_i t}\}, \tag{82}$$

where $\gamma_i(t) = \sqrt{\frac{\sigma_i}{v_i(t)(1+\theta_i(t))}}$, $\delta_i(t) = \sqrt{\frac{\beta_i}{v_i(t)(1+\theta_i(t))}}$, $0 < \sigma_i < 1$, $\beta_i > 0$, $\theta_i(t) = \bar{c}_i(t) + \sum_{j=1}^N a_{ij}\bar{c}_j(t)$, $\dot{v}_i(t) = e^{2\lambda t}(1 + \theta_i(t))\,\|\bar{x}_i(t)\|^2$, and $v_i(0) > 0$. The fully distributed adaptive event-triggered controller based on the edge information is given as follows [163]:

$$\begin{aligned} u_i(t) &= K\sum_{j=1}^{N} c_{ij}(t)a_{ij}\hat{z}_{ij}(t), \\ \dot{c}_{ij}(t) &= \kappa_{ij}a_{ij}[-\rho_{ij}c_{ij}(t) + \hat{z}_{ij}^T(t)\Gamma\hat{z}_{ij}(t)], \\ \hat{z}_{ij}(t) &= e^{A(t-t_k^i)}x_i(t_k^i) - e^{A(t-t_{k'}^j)}x_j(t_{k'}^j), \end{aligned} \tag{83}$$

and the adaptive triggering mechanism is given as

$$\begin{aligned} t_{k+1}^i = t_k^i + \inf_{t>t_k^i}\Bigg\{t - t_k^i \Bigg| &\sum_{j=1}^{N}(1+\delta_{ij}c_{ij}(t))\|e_{xi}(t)\|^2 \\ &\geq \frac{1}{4}\sum_{j=1}^{N} a_{ij}\hat{z}_{ij}^T(t)\Gamma\hat{z}_{ij}(t) + \mu e^{-\nu t}\Bigg\}, \end{aligned} \tag{84}$$

where $\mu > 0$ and $\nu > 0$. It should be noted that under the proposed control strategies, the prior knowledge of global graph information can be avoided.

More recently, cooperative control problems of heterogeneous MASs have been studied by using ETC strategies [164–172]. Based on the distributed observer and the combinational measurement approach, a distributed ETC strategy is developed in [164], and is given as follows:

$$\begin{aligned} u_i(t) &= K_{1i}\hat{x}_i(t) + K_{2i}\hat{v}_i(t), \\ \dot{\hat{v}}_i(t) &= A_0\hat{v}_i(t) + K_\eta\Bigg(\sum_{j\in\mathcal{N}_i}(\hat{v}_j(t_k^i) - \hat{v}_i(t_k^i)) \\ &\quad + a_{i0}(v(t_k^i) - \hat{v}_i(t_k^i))\Bigg), \\ \dot{\hat{x}}_i(t) &= A_i\hat{x}_i(t) + B_iu_i(t) + E_i\hat{v}_i(t) \\ &\quad + H_i(C_{mi}\hat{x}_i(t) + D_{mi}u_i(t) + F_{mi}\hat{v}_i(t) - y_{mi}(t)), \end{aligned} \tag{85}$$

where $\hat{v}_i(t)$ is the state of the distributed observer. Define the measurement errors as

$$e_{qi}(t) = q_i(t_k^i) - q_i(t), \quad e_{pi}(t) = p_i(t_k^i) - p_i(t), \tag{86}$$

where $q_i(t) = \sum_{j\in\mathcal{N}_i}(\hat{v}_j(t) - \hat{v}_i(t))$ and $p_i(t) = v(t) - \hat{v}_i(t)$. Then, the event-triggering mechanism is designed as follows:

$$t_{k+1}^i = t_k^i + \inf_{t>t_k^i}\{t - t_k^i | \bar{e}_{qi}(t) > \beta_i\bar{q}_i(t)\}, \tag{87}$$

where $\bar{e}_{qi}(t) = \sqrt{\|e_{qi}(t)\|^2 + a_{i0}^2\|e_{pi}(t)\|^2}$, $\bar{q}_i(t) = \|q_i(t)\| + a_{i0}\|p_i(t)\|$ and $\beta_i > 0$. Under the proposed ETC strategy, the output of each agent tracks the output of the leader asymptotically. The authors in [165,166] propose adaptive ETC strategies for heterogeneous MASs without and with uncertainties, respectively. Based on the internal model principle, the authors in [167] present a practical ETC strategy for a class of minimum-phase uncertain heterogeneous MASs.

In this section, various event-triggering mechanisms and ETC strategies are reviewed. It should be noted that there are two main issues in event-triggered cooperative control, that is, continuous monitoring of measurement errors and Zeno behavior. For the issue of Zeno behavior, some advanced event-triggering mechanisms are reviewed. For the issue of continuous monitoring, advanced event-triggering mechanisms based on the open-loop estimation approach and self-triggering mechanisms are reviewed. Moreover, on the basis of the above mentioned mechanisms, the studies of adaptive event-triggering mechanisms are reviewed.

6. Conclusions

This paper presents a brief survey on recent advances of distributed coordination of MASs. The particular attention is given to cooperative control of MASs with various agent dynamics, cooperative control of MASs with communication delays and event-triggered cooperative control of MASs.

In cooperative control of MASs with various agent dynamics, we focus on consensus of homogeneous and heterogeneous linear MASs, COR of linear MASs, cooperative control of MASs with unknown agent dynamics and cooperative control of MASs with nonlinear agent dynamics. In addition, we also touch formation control and containment control.

In cooperative control of MASs with communication delays, approaches to convergence analysis and controller synthesis of MASs with bounded communication delays based on Lyapunov–Razumikhin functions, Lyapunov–Krasovskii functionals, the frequency domain method, the contraction method are first reviewed. Some more recent results on cooperative control of MASs with unbounded communication delays are also reviewed.

In event-triggered cooperative control of MASs, various event-triggering mechanisms and corresponding ETC strategies are reviewed. Moreover, two key issues in event-triggered cooperative control of MASs, that is, continuous monitoring and Zeno behavior have been highlighted.

It can be observed that although significant advances have been achieved in distributed coordination of MASs, there are still many interesting, important and challenging problems deserving further investigation. Some of them are listed as follows:

- *Cooperative control with infinite transmission delays under more general scenarios*. It should be noted that the existing works [126,129] focus on cooperative control of MASs with unbounded distributed transmission delays only under fixed communication topology. However, communication topologies among agents may change dynamically when communication links experience failures or creations due to external disturbances or limited communication/sensing ranges. Thus, it would be an interesting topic to consider cooperative control of linear MASs with unbounded distributed transmission delays under switching topologies. On the other hand, the existing works [130,124], only address cooperative control of single-integrator MASs with unbounded time varying transmission delays. It is desirable to consider MASs with more general agent dynamics and unbounded time-varying delays.

- *Cooperative control of MASs with more general nonlinear dynamics*. It is noted that the existing results on cooperative control of nonlinear MASs are quite restrictive as only very special nonlinear agent dynamics are considered. In practice, agent dynamics are often more general and it is thus desirable to study cooperative control of MASs with more general nonlinear agent dynamics.
- *Performance of event-triggered cooperative control*. As mentioned in many literatures, compared with conventional periodic sampling control, ETC can improve the efficiency of resource utilization. However, to the best of our knowledge, the quantitative performance comparison between ETC and periodic sampling control in the context of MASs is still an open problem. To evaluate how much energy or bandwidth can be saved while guaranteeing a certain control performance, it is critical to define some proper quantitative performance indexes. Then with such indexes, fair and more convincing comparison can be made between ETC strategies and periodic sampling control strategies.

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